"*Whatever it takes*" Is All You Need: Monetary Policy and Debt Fragility*

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Abstract

The valuation of government debt is subject to strategic uncertainty. Pessimistic lenders, fearing default, bid down the price of debt, leaving a government with a higher debt burden. This increases the likelihood of default, thus confirming the pessimism of lenders. Can monetary interventions mitigate debt fragility? With one-period commitment to a state contingent policy, the monetary authority can indeed overcome strategic uncertainty. Under discretion, debt fragility remains unless reputation effects are sufficiently strong. Simpler forms of interventions, such as an inflation target, cannot eliminate debt fragility.

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JEL classification: E42, E58, E63, F33.

1 Introduction

> *But there is another message I want to tell you. Within our mandates, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.* [Mario Draghi, July 2012][1]

This paper studies the interaction of fiscal and monetary policy in the presence of strategic uncertainty over the value of government debt. In real economies, beliefs of investors about the likelihood of government default,

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[1] This statement is an excerpt from the address of Mario Draghi, President of the European Central Bank, at a financial conference, in July 2012.

and hence the value of its debt, can be self-fulfilling. Pessimistic investors, fearing government default, will only purchase government debt if there is a sufficient risk premium. The resulting increase in the cost of funds makes default more likely.[2] Pessimism can be self-fulfilling even if fundamentals are sound enough that an equilibrium without default exists as well.

These results hold for real economies, in which the intervention of a monetary authority is not considered. Does debt fragility exist in a nominal economy? The presence of a monetary authority can provide an alternative source of revenue through an inflation tax and perhaps use its influence to stabilize real interest rates. Can the monetary authority act to eliminate strategic uncertainty over the value of sovereign debt? If so, will it have an incentive to do so? The answers to these questions are relevant for assessing appropriate monetary interventions to counter strategic uncertainty in debt markets.

This emphasis on strategic uncertainty was also recognized by Mario Draghi, President of the European Central Bank (ECB). Later, in September 2012, he said:[3]

> *...the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios...*

The overlapping generations model with fiscal and monetary interventions provides a framework for analysis. The model is structured to highlight strategic uncertainty in the pricing of sovereign debt stemming from the default choice of a government. By construction, there is an equilibrium without default, and in general there are other equilibria with state contingent default.

The institutional structure of the model is intended to capture key features of the European experience. The sequence of events in the eurozone is consistent with the view that the ECB's pledge to undertake any policy measure to anchor interest rates on a *"good equilibrium"* was critical to counter pessimistic investor's sentiment. This contrasts with repeated failed attempts of national fiscal authorities to convey any credible plan and contain confidence driven swings in sovereign debt price. By design, our model does not allow any commitment by fiscal authorities to the repayment of debt and closely links monetary policy to an inflation target, consistent with the principal mandate of the ECB. Thus the analysis follows the eurozone institutional structure and investigate whether central bank's credibility is effective to deter self-fulfilling crises, when fiscal authorities are restricted to sequential choices given market conditions.[4]

The monetary authority intervenes through transfers to the fiscal entity, financed by an inflation tax. The monetary intervention has a number of influences. First, the inflation tax delivers real resources to the government, thus reducing the distortions from taxation. Second, the realized value of inflation alters the real value of debt and consequently the debt burden left to the fiscal authority. Third, it may impact expectations of future inflation and thus money demand and nominal interest rate. Given these transfers and its outstanding obligations, the fiscal

[2] This interaction between beliefs and default is central to Calvo (1988); and other contributions that followed, including Cole and Kehoe (2000), Roch and Uhlig (2016) and Cooper (2012).

[3] This quote is taken from `http://www.ecb.europa.eu/press/pressconf/2012/html/is120906.en.html`.

[4] Our analysis focuses on how central bank's announcements can credibly stabilize sovereign debt market, as the European experience suggests. Still, interventions other than this particular ECB pledge took place contemporaneously, such as improved supervision of banks and programs by the European Stability Mechanism, which surely impacted market conditions as well. Further, as emphasized by a referee, a **credible** commitment to bailout would have similar effects. In this context though, the credibility of a bailout is dependent upon the size of government debt.

authority chooses to default or not. Our analysis emphasizes the dependence of this default decision, and thus the extent of strategic uncertainty, on the conduct of monetary policy.

The paper is constructed around two polar cases, distinguished by the ability of the monetary authority to commit and the complexity of its intervention. In the first case, studied in Section 3, monetary policy decisions are made by an independent central bank with the ability to commit.

One leading example of monetary policy under commitment is the implementation of a strict inflation target, a relatively common form of monetary rule. As the central bank is bound to deliver an unconditional inflation rate, it has no flexibility to respond to variations in sentiment: strategic uncertainty in the valuation of debt remains in this monetary economy.

However, there is a more nuanced monetary intervention that can eliminate debt fragility. Under the policy we design, the central bank uses its commitment power to stabilize sovereign debt valuations. Interestingly, this desired intervention does not "bail-out" the fiscal authority. Rather, the countercyclical nature of the policy induces an accommodative fiscal stance only in times of low productivity. In effect, the intervention relies on the unique capacity of the central bank to turn a non state contingent nominal asset into a real state contingent asset, by distributing the effects of inflation across states.[5] Overall, this policy rule leans against negative sentiments of investors and preserves the fundamental price of debt. Further, it does not endanger the primary objective of the central bank, to anchor inflation expectations around an inflation target, as in its mandate.

This particular intervention is effective as an off-equilibrium threat, where its mere announcement is enough to stabilize debt valuations. It is a representation of the ECB's pledge to intervene, reflected in the above quote of Mario Draghi. We denote this policy *"wit"*, to follow his statement to do *"**w**hatever **i**t **t**akes"* to counter pessimistic self-fulfilling expectations in eurozone sovereign debt markets.[6]

But absent a mechanism for commitment, is this policy credible? This brings us to a second case in which the monetary authority operates under discretion, and thus takes decisions sequentially, possibly building on its reputation. Consistent with our institutional set-up, the analysis is asymmetric, allowing reputation to matter for monetary but not fiscal entities. This reflects our interest in understanding how the monetary authority can cope with debt fragility rather than the sources of that fragility. Given the state of the economy, money creation and labor taxes are set to minimize tax distortions, keeping in mind the impact of these choices on the central bank's reputation. Default is also chosen optimally. In equilibrium, these fiscal and monetary choices determine inflation expectations.

If reputation effects are strong enough, policy *"wit"* is credible, so that commitment to its implementation is not needed. The implementation is built upon a punishment: a deviating monetary authority returns to a strict inflation target regime, which is the institutional foundation of many modern central banks, including the ECB. Not surprisingly, all else the same, a patient monetary authority is less likely to deviate. But there is another element in the analysis: the higher the risk of self-fulfilling debt crisis in the inflation target regime, the **more** credible is

[5]The capacity of the monetary authority to stabilize sovereign debt markets relies on the interplay of state dependent inflation and associated expectations, not on on the collection of revenue from the inflation tax. The benefit of flexible monetary policy rules have been outlined for instance by Calvo and Guidotti (1993) in the context of an optimal taxation program: the inflation tax should absorb all source of randomness, i.e. be countercyclical. Our analysis stresses an additional benefit of such rules, namely the possibility to stabilize sovereign debt markets.

[6]The design of *sophisticated* monetary policies to enforce a unique competitive equilibrium is the focus of Atkeson, Chari, and Kehoe (2010). Our analysis develops a similar mechanism in the context of self-fulfilling debt crisis with an additional emphasis on credibility.

the promise of the central bank to intervene *ex post* to counter pessimistic beliefs on debt valuation. Evidently, the combination of a patient central bank and large enough strategic uncertainty in the absence of intervention supports a desired outcome.

Alternatively, a very impatient monetary authority, operating under complete discretion, will yield to the temptation of using a non-distortionary inflation tax to provide a bailout and finance fiscal needs. We show that this policy does not eliminate debt fragility. Indeed, our nominal environment captures the expectation reaction to monetary discretion: anticipating monetary financing, private agents adjust their demand for money and investors charge a higher nominal interest rate to make up for expected inflation. In equilibrium, self-fulfilling debt crisis goes hand in hand with self-fulfilling inflation.

Other analyses examine possible strategies for central banks to address self-fulling debt crises. Calvo (1988) extends his real economy to include a discussion of inflation as a form of partial default. He argues that there may exist multiple equilibria in the determination of inflation and the nominal interest rate on government debt. For this analysis, there is no interaction between fiscal and monetary debt repudiation.

Corsetti and Dedola (2016) augments Calvo's framework to study the interaction of fiscal and monetary policy. Their analysis retains some of the central features of Calvo's model, and argues that discretionary monetary interventions through the printing press will not generally resolve debt fragility. But, the central bank, through its holding of government debt, can have a stabilizing influence.

Aguiar, Amador, Farhi, and Gopinath (2013) build a nominal economy with debt roll-over crisis, as in Cole and Kehoe (2000). They investigate the optimal degree of conservativeness of the central bank (as in Rogoff (1985)) as a tool to address inefficient debt crises. Moderate inflation aversion contains the occurrence of self-fulfilling debt crisis and restrains the inflation bias in normal times.[7]

Our analysis differs from these papers in a couple of fundamental ways. First, our emphasis is on monetary and fiscal institutional interactions. While the above mentioned papers study economic outcome under policy discretion, we analyze how different monetary policy regimes (and their credibility) influence the fragility of sovereign debt markets. In other terms, we unveil how a central bank can build upon its reputation to anchor inflation expectations and provide a credible backstop to sovereign debt.

Especially, in contrast to Aguiar, Amador, Farhi, and Gopinath (2013) and Corsetti and Dedola (2016), our paper finds conditions for state contingent monetary interventions to eliminate debt fragility. The stabilizing policy anchors inflation expectations and is anticipated to respond to strategic uncertainty in a manner that deters state contingent default. In equilibrium, these interventions are never observed and debt markets are stable. When policymakers are sufficiently patient and/or strategic uncertainty is severe enough, this outcome can be supported without commitment. The value of maintaining a reputation allows the central bank to both stabilize debt markets and anchor inflation expectations. If reputation costs are not strong enough, the monetary authority is doomed to generate high inflation on top of self-fulfilling debt crisis. The analysis thus makes clear how a high cost of generating surprise inflation, through a reputation loss, anchors inflation expectations, disciplines the monetary authority, and makes interventions that stabilizes debt markets credible.

Finally, money demand is endogenous in our model, derived from household intertemporal optimization. This

[7]Relatedly, Bacchetta, Perazzi, and van Wincoop (2018) evaluate quantitatively the credibility of monetary interventions to deter interest rate accumulation and self-fulfilling default, in a New Keynesian environment with "slow-moving" debt crisis, as in Lorenzoni and Werning (2013).

creates a complementarity between expected and realized inflation. In equilibrium, both real economic activity and money demand reflect expected inflation. This link from anticipations about monetary policy to real decisions is a novel feature of our model relative to these other papers and has a direct influence on the characterization of equilibria under discretion.

The paper is structured as follows. Section 2 describes the economic environment and the fiscal problem of the government. The equilibrium concept is defined here as well. Section 3 investigates the presence of debt fragility under monetary commitment and considers two policies: (i) a strict inflation target and (ii) *"whatever it takes"*. Section 4 characterizes equilibria in a regime without monetary commitment. It presents conditions such that policy *"wit"* is credible. It further characterizes the outcome of pure discretion in the absence of any reputation effects, leading to both debt fragility and self-fulfilling inflation. Section 5 concludes.

2 Economic Environment

Consider an overlapping generation economy with domestic and foreign agents. Agents live two periods. Time is discrete and infinite. Agents differ in productivity in young age and this generates different asset demands. Relatively poor agents hold money as a store of value while richer agents incur a cost to obtain intermediated claims with higher returns. The government, composed of a central bank and a treasury, issues debt each period and faces a choice on how to finance the repayment of its obligations. In particular, the government can tax labor income, print money or default on its debt.

In this section, we describe the choices of private agents, the fiscal environment and the institutional framework for the conduct of monetary policy. The section also includes a definition of equilibrium.

2.1 Private Agents

Every period, a continuum of mass 1 of domestic agents (households) is born and lives two periods. These agents produce a perishable good in both young and old age. They consume only when old. Limited financial market participation sorts agents in two groups. For convenience, we will refer to *poor* agents, who will hold only money in equilibrium, and *wealthy* agents, who hold intermediated claims.

Production is linear. In youth, productivity is heterogenous. A mass ν^p of *poor* agents have low productivity $z^p = 1$. A mass $\nu^w = 1 - \nu^p$ of *wealthy* agents have high productivity $z^w = z > 1$. In old age, productivity A is stochastic, iid, and common to all old agents.[8] Access to intermediated claims is costly: agents pay a participation cost Γ to invest in nominal government bonds or in a risk-free asset, e.g. storage, that delivers a real return $R > 1$.

2.1.1 Poor Households

Poor households have low labor productivity $z^p = 1$ in youth. Their savings between young and old age are composed only of money holdings, whose real return is given by $\tilde{\pi}'$, the inverse of the gross inflation rate.[9] Poor

[8] Formally, the distribution of A has full support on the closed and compact set $[A_l, A_h]$. $F(\cdot)$ is the associated cumulative distribution function, and $f(\cdot) = F'(\cdot)$.

[9] We verify later that these agents prefer to save via money rather than costly intermediated claims in equilibrium.

households solve:

$$\max_{n,n',c'} \; E\big[u(c') - g(n')\big] - g(n), \tag{1}$$

subject to young and old age real budget constraints:

$$m = n \tag{2}$$

and

$$c' = \begin{cases} A'n'(1-\tau') + m\tilde{\pi}' + t' & \text{if} \quad D = r \qquad (a) \\ A'n'(1-\gamma) + m\tilde{\pi}' + t' & \text{if} \quad D = d \qquad (b) \end{cases} \tag{3}$$

In youth, poor agents supply labor n and have real money holdings, m, carried on from young to old age. Return on money is given by the gross inverse inflation rate $\tilde{\pi}'$. In old age, poor agents supply labor n', which is augmented by aggregate productivity A'. Old age budget constraints (3)(a) and (3)(b) reflect the influence of fiscal choices on production and consumption, as detailed in Section 2.2. For now, note that in case of debt repayment ($D = r$), τ' is the tax rate on labor income of old agents; in case of default ($D = d$), γ is a productivity loss. Finally, $t' \geq 0$ is a lump-sum transfer. To be clear, here and below both $\tilde{\pi}'$ and t' may depend on D too.[10] Denote by n_y^p and n_o^p the optimal labor supply decision of young and old poor agents.

The analysis imposes the following utility function: $u(c) = c$ and $g(n) = \frac{n^2}{2}$. The linear quadratic structure is introduced to neatly capture the reaction of agents to government policy choices.[11] With these preferences, labor supply decisions are:

$$n_y^p = E(\tilde{\pi}') \tag{4}$$

in youth and

$$n_o^p = \begin{cases} A'(1-\tau') & \text{if} \quad D = r \qquad (a) \\ A'(1-\gamma) & \text{if} \quad D = d \qquad (b) \end{cases} \tag{5}$$

in old age.

Labor supply in both young and old age are driven by real returns to working. In youth, agents form expectations $\tilde{\pi}^e = E(\tilde{\pi}')$, and supply labor accordingly: if agents expect high inflation, i.e. a low $\tilde{\pi}^e$, they will reduce labor supply and the associated demand for real money holding. Similarly, either taxes or the default cost imposed on old age labor income are distortionary, a high tax rate or default cost reduces the return to working and hence old agents' labor supply.

In contrast to, for example, Calvo (1988), money demand is endogenous in our model, reflecting labor supply and asset market participation decisions. Consequently, expected monetary interventions can influence the magnitude of the *ex post* tax base created by money holdings, a key element of equilibria arising under commitment and discretion.

[10] We specify later how these policy objects depend on the default decision when introducing monetary regimes.

[11] The risk neutrality of agents eliminates risk sharing from the model, thus allowing to focus on the key issue of the efficiency of labor and inflation taxes. We comment in Section 4.1 and Appendix 6.8 on some policy implications of risk aversion.

2.1.2 Wealthy Households and Financial Intermediation

Wealthy households differ from poor agents by their productivity in youth, $z^w = z > 1$. This higher productivity induces them to pay the fixed cost Γ to access intermediated saving. A parametric restriction ensures that young wealthy agents save via the financial sector for any positive expected inflation rate.[12] Formally,

Assumption 1.

$$z^2 > \frac{R\Gamma}{R^2 - 1} > 1. \tag{\textbf{A.1}}$$

The wealthy solve:

$$\max_{n,n',c'} E\big[u(c') - g(n')\big] - g(n), \tag{6}$$

subject to young and old age real budget constraints:

$$m + b^w + k = zn - \Gamma \tag{7}$$

$$c' = \begin{cases} A'n'(1-\tau') + \tilde{\pi}'m + (1+i')\tilde{\pi}'b^w + Rk + t' & \text{if} \quad D = r \qquad (a) \\ A'n'(1-\gamma) + \tilde{\pi}'m + Rk + t' & \text{if} \quad D = d \qquad (b) \end{cases} \tag{8}$$

In youth, wealthy agents supply labor n and produce zn. After incurring the fixed cost Γ, they invest a per capita amount b^w in government bonds and k in risk-free assets. Government debt is nominal and pays an interest rate i' next period if there is no default. When old, these agents supply labor n' and consume c', contingent on the realization of A' and the fiscal choices of the government.

Finally, given linear utility of consumption, the portfolio decision between intermediated saving $b^w + k$ and money holding m is only driven by expected returns. Under Assumption 1, as long as expected return on money holding $\tilde{\pi}^e$ is strictly inferior to the real return R on the risk-free asset, these rich households do not hold money. The portfolio for intermediated savings will include both nominal government debt and risk-free asset as long as the expected return on government debt equals that on the asset:

$$(1+i')\int_{A'} \tilde{\pi}' \mathbb{1}_D dF(A') = R, \tag{9}$$

where $\mathbb{1}_D$ is an indicator equals to 1 if the treasury repays debt, and 0 otherwise. Here the expectation is over the future value of A', since both the default and inflation decisions may depend on its realization. We refer to this pricing equation as the "no-arbitrage condition".

Denote by n_y^w and n_o^w the optimal labor supply decisions of wealthy agents in young and old age. The solution to (6) with $u(c) = c$ and $g(n) = \frac{n^2}{2}$ implies:

$$n_y^w = Rz \tag{10}$$

[12] We verify this in characterizing equilibria. Note that a model with a continuum of agents, as in Freeman and Huffman (1991), would endogenize the distribution of *poor* and *wealthy* agents. As argued in Section 3.2, our results are not sensitive to this modeling approach.

in youth and

$$n_o^w = \begin{cases} A'(1-\tau') & \text{if} \quad D = r \qquad (a) \\ A'(1-\gamma) & \text{if} \quad D = d \qquad (b) \end{cases} \tag{11}$$

in old age. Labor supply n_y^w of young agents is determined by the expected return R on intermediated savings. In old age though, the effective return on debt will depend on the realized inverse inflation rate $\tilde{\pi}'$, the nominal interest rate i' and the default decision of the government.

2.1.3 Foreign Households

As is traditional in the sovereign debt literature, in addition to domestic agents, there are also foreign households who hold domestic debt. This feature of the model ensures that our results are not sensitive to a specific pattern of holding of public debt. Foreign households are risk neutral and have access to certain return of R as an alternative store of value. In equilibrium, they hold a fraction $(1-\theta)$ of domestic debt.[13]

2.2 The Government

The government is composed of a treasury and a central bank. Every period, it has to finance a constant and exogenous flow of real expenses g. Government expenditures do not enter into agents utility. To finance these expenses, it issues nominal debt, with real value of b so that $b = g$. The government uses revenue from labor taxes and seignorage from printing money to repay principal and interests on debt.[14] Alternatively, the treasury can default on its debt obligations. The decisions of both the treasury and the central bank are made with the objective of maximizing the welfare of domestic private agents.[15]

2.2.1 Central Bank

The monetary authority is independent of the treasury. The analysis characterizes equilibria for various types of monetary interventions, dependent upon the ability of the central bank to commit to its policy. Specifically, the analysis first considers a regime in which the central bank can commit to state contingent policies. This includes an inflation target regime and our version of the ECB's pledge to implement *"whatever it takes"*. This analysis is followed by consideration of monetary policy without commitment.

For each case, the monetary action is essentially a state contingent transfer made to the treasury. In each period, the treasury takes as given the transfer from the central bank and associated inverse inflation rate $\tilde{\pi}$. It optimally chooses whether or not to default and, in the event of repayment, the labor tax rate. The monetary authority recognizes its influence on these decisions.

2.2.2 Treasury

By assumption, fiscal policy is generation specific. A given fiscal authority can only tax members of the current old

[13]Given the indifference of risk neutral agents regarding their portfolio of government debt and storage, θ is not determined in equilibrium. Thus equilibrium will be characterized for given values of θ.

[14]The assumption that new expenses are financed exclusively by new debt allows us to focus on how the debt is repaid rather than its magnitude.

[15]As such, unlike other papers in this literature, there is no additional *ex post* cost of inflation beyond that induced by the utility of private agents. We discuss how adding this additional feature would alter our findings in Section 4.1.

generation. Intergenerational transfers are not feasible. This use of generational budget balance appears in Chari and Kehoe (1990) and Cooper, Kempf, and Peled (2010), for example. With this institutional set-up, the analysis focuses on the choice between taxation and default. It is consistent with our emphasis on the role of monetary interventions on sovereign debt market's fragility.[16]

Under repayment, and given transfers from the monetary authority, the real budget constraint of the treasury is:

$$(1+i)\tilde{\pi}b = \tau\big(\nu^p A n_o^p(\tau) + \nu^w A n_o^w(\tau)\big) + \frac{\Delta M}{P}. \tag{12}$$

The left hand side represents the real liabilities of the government, net of realized inverse inflation rate $\tilde{\pi}$, where $b = g$ is real debt outstanding. On the right hand side, $n_o^j(\tau)$ is the labor supply decision of old agents of type $j \in \{p, w\}$, ΔM is the change in the total money supply (M) and P is the price level. Denote by σ the rate of money creation that implements the change in money supply ΔM.

Instead of repayment, the treasury can fully renege on its debt. But there are two costs of default for domestic agents. First, direct costs of default are born by old wealthy agents, who hold a fraction θ of government debt. Second, if the treasury repudiates its debt, the country suffers from a deadweight loss, as commonly assumed in the literature on strategic default.[17] Formally, aggregate productivity contemporaneously drops by a proportional factor γ. The model excludes punishments involving exclusion from future capital markets. This reflects the quantitative finding that the main force preventing default is the direct output loss.[18]

As the government budget constraint holds over time for a given generation, a decision of the treasury to default on period t debt has no direct effect on future generations. That is, default affects only the welfare of current old agents, who otherwise are taxed via seignorage or labor tax.[19] Given the decision of the central bank, the treasury weighs the welfare burden of tax distortions against the direct costs and penalty induced by the default decision. Denote by $W^r(\cdot)$ the welfare of the economy under *repayment* and by $W^d(\cdot)$ under *default*. The decision to default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

Given aggregate productivity A, nominal interest rate i, real money tax base m_{-1}, money printing rate σ and the induced inverse inflation rate $\tilde{\pi}$, the welfare criterion of the treasury $W^D(\cdot)$ for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^p\Big(c_o^p(D) - \frac{n_o^p(D)^2}{2}\Big) + \nu^w\Big(c_o^w(D) - \frac{n_o^w(D)^2}{2}\Big). \tag{13}$$

As above, the levels of $\tilde{\pi}$ are implemented under each option, as a function of the monetary regime under which

[16]Debt rollover to spread repayment burden across generations is not allowed. As stressed in Lorenzoni and Werning (2013), the ability of the government to rollover debt is limited because there is an ultimate choice of whether to repay or default. Our focus is on that choice.

[17]Penalties and direct sanctions are central theoretic concepts for enforcement of international asset trade. See the seminal work by Eaton and Gersovitz (1981). For an extensive review, see Eaton and Fernandez (1995).

[18]Empirical evidence regarding reputation costs of default are mixed: exclusion from international credit markets are short-lived and premium following defaults are usually found to be negligible. An extensive discussion can be found in Trebesch, Papaioannou, and Das (2012). From a theoretical point of view, Bulow and Rogoff (1989) show that reputation mechanisms cannot enforce international asset trade, if the government can buy foreign assets as an alternative source of insurance.

[19]The assumption of no taxation of income when young is just a simplification that allows us to neatly disentangle demand for money, for intermediated claims and labor supply driven by taxation.

the economy operates. Specifically, under repayment, $D = r$, the welfare of old agents is:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^p m_{-1}\tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^w R(Rz^2 - \Gamma). \tag{14}$$

where τ clears the budget constraint (12). Here inflation is created by the printing of money that is transferred directly to the treasury.

The option to default, $D = d$, triggers penalties but no tax need be raised. In keeping with the generational view of the budget constraint, any money creation in the current period is transferred lump-sum to the current old. The amount of this transfer will depend on the monetary regime. In this case, the welfare of old agents becomes:

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^p m_{-1}\tilde{\pi}^d - R\theta b + \nu^w R(Rz^2 - \Gamma) + T(\sigma, m_{-1}, \tilde{\pi}^d), \tag{15}$$

where $T(\sigma, m_{-1}, \tilde{\pi}^d)$ is the aggregate lump sum transfer to old agents that implements $\tilde{\pi}^d$.[20]

As made clear in expressions (14) and (15), the share θ of debt held by domestic agents influences the welfare criteria of the fiscal authority, and eventually its policy choices. This modeling feature guarantees that our results, particularly multiplicity of debt values, are not sensitive to the national identity of debt holders.

2.3 Assumptions

The following two assumptions are used for characterizing equilibria. The first places a lower bound on γ so that default is costly, especially when no debt is held by domestic agents.

Assumption 2.

$$\frac{A_l^2\gamma(2-\gamma)}{2} > \nu^p. \tag{\textbf{A.2}}$$

Under this assumption, default is not a desirable option when seignorage revenue alone could service principal and interest on debt.[21]

The next assumption ensures that the fundamentals of the economy are compatible with a risk-free outcome, i.e. given the real level of debt b and a real interest rate R, debt will be repaid for all A. As demonstrated below, this implies there will be a solution to (9) without default, under the monetary regimes we study. Formally,

Assumption 3. $b < \bar{b}$ *where*

$$\bar{b} = \frac{A_l^2(1-\gamma)\gamma}{R}. \tag{\textbf{A.3}}$$

Note that Assumption 3 is stated in the extreme case where there is no seignorage revenue, and all debt held by foreigners.[22] The presence of an equilibrium without default provides a convenient benchmark for the analysis.

[20] Computations to derive (14) and (15) are detailed in Appendix 6.1.

[21] This is established in the construction of equilibria.

[22] To derive this restriction, combine the decision of the treasury to repay, (14) and (15), with the government budget constraint with no inflation, no fiscal revenue from seignorage and all debt held by foreigners ($\theta = 0$). It implies that there will be an equilibrium without default risk when some of the debt is held by domestic agents and when money printing does provide resources to the fiscal authority. Indeed, domestic holding of public debt or a higher money printing rate relaxes the willingness of the fiscal authority to default rather than repay its debt.

Assumptions 1-3 are maintained throughout the rest of the analysis. At appropriate points, we discuss their importance of our results.[23]

2.4 State Variables and Equilibrium Definition

Our analysis investigates whether monetary interventions can deter self-fulfilling variations in sovereign debt price. The source of strategic uncertainty is grounded in the multiple solutions to the no-arbitrage condition on debt, (9). This is made explicit in sub-section 3.1. The conduct of monetary policy influences the nominal interest rate i directly via inflation expectations, and also via its influence on the treasury's decision to repay or default on debt.

Strategic uncertainty is modeled through a sunspot variable, denoted $s \in \{s^o, s^p\}$, that corresponds to confidence of domestic and foreign households about the repayment of government debt next period. If $s = s^o$, agents are "optimists" : they coordinate on the risk-free (fundamental) price of the government debt. If $s = s^p$, agents are "pessimists" : they coordinate on a higher risk / lower price equilibrium with state contingent default. In the event there is a unique equilibrium price, then the fundamental price obtains regardless of the sunspot realization. The distribution of sunspot shocks is i.i.d. Denote by $p_o \in (0, 1)$ the probability of optimism, i.e. $s = s^o$.

The state of the economy is $\mathcal{S} = (A, i, m_{-1}, s, s_{-1})$. Aggregate productivity, A, is realized and directly affects the productivity of the old. There are two endogenous predetermined state variables, m_{-1} and i, respectively real money holdings of current old agents, and the nominal interest rate on outstanding public debt. Both the sunspot shock last period, s_{-1}, and the current realization, s, may impact fiscal policy, monetary policy and the choices of private agents.

To define a Stationary Rational Expectations Equilibrium (SREE), it is necessary to define market clearing conditions and the link between money printing, inflation and seignorage revenue.

2.4.1 Market Clearing

In every state, the markets for money and bonds must clear. The condition for money market clearing is

$$\nu^p m(\mathcal{S}) = \frac{M(\mathcal{S})}{P(\mathcal{S})} \qquad \forall \mathcal{S}, \tag{16}$$

where $P(\mathcal{S})$ is the state dependent money price of goods and $M(\mathcal{S})$ is the stock of fiat money. This equation implies that young agents' real money demand equals the real value of the supply.

The market for government debt clears if the no-arbitrage condition (9) holds and the savings of wealthy households plus the demand from foreigners is not less than the real value of newly issued debt. We assume that foreigners' endowment is large enough to clear the market for bonds as long as (9) is met.

2.4.2 Government Budget Constraint, Inflation and Seignorage

The SREE version of the government budget constraint, (12), requires a couple of building blocks. The inverse inflation rate, $\tilde{\pi}$, is given by:

$$\tilde{\pi}(\mathcal{S}) = \frac{P(\mathcal{S}_{-1})}{P(\mathcal{S})} = \frac{m(\mathcal{S})}{m(\mathcal{S}_{-1})} \frac{1}{1 + \sigma(\mathcal{S})}, \tag{17}$$

[23] There is a final assumption about the conduct of monetary policy under discretion. It is stated in the context of that monetary regime in Section 4.

using (16). Revenue from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \sigma(\mathcal{S})\nu^p m(\mathcal{S}_{-1})\tilde{\pi}(\mathcal{S}) = \nu^p m(\mathcal{S})\left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})}\right). \tag{18}$$

Here $m(\mathcal{S}_{-1})$ represents the real money holdings of the current old. Importantly, these equations imply a one-to-one mapping between the rate of money creation $\sigma(\mathcal{S})$ and realized inverse inflation $\tilde{\pi}(\mathcal{S})$. This reflects the fact that $m(\mathcal{S}_{-1})$ is predetermined and that money demand for the current generation, $m(\mathcal{S})$, is, as we verify below, independent of the current rate of money creation. Accordingly, the equilibrium definition is stated with the government setting inflation $\tilde{\pi}(\mathcal{S})$.[24]

Substituting these expressions for seignorage and the inverse inflation rate into (12), we can write the government budget constraint as:

$$(1+i)\tilde{\pi}(\mathcal{S})b = A^2\big(1-\tau(\mathcal{S})\big)\tau(\mathcal{S}) + \nu^p m(\mathcal{S})\left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})}\right). \tag{19}$$

As alternative regimes of monetary interventions are presented, the determination of $\tilde{\pi}(\mathcal{S})$ and hence $\sigma(\mathcal{S})$ is made explicit.

2.4.3 Stationary Rational Expectations Equilibrium

Our analysis is organized around different institutional arrangements that determine the influence of monetary policy on fiscal decisions and asset prices. To do so, we contrast economic outcomes within a common equilibrium definition.

Definition 1. *A Stationary Rational Expectations Equilibrium (SREE) is given by:*

1. *Labor supply and saving decisions of private agents,* $\big(n_y^p(\mathcal{S}), n_o^p(\mathcal{S}), n_y^w(\mathcal{S}), n_o^w(\mathcal{S}), m(\mathcal{S}), k(\mathcal{S}), b(\mathcal{S})\big)$*, who form rational expectations in youth, supply labor in young and old age, solve* (1) *and* (6) *subject to their respective budget constraints* (2)*,* (3)*,* (7) *and* (8)*, given state contingent monetary and fiscal policies* $(\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S}))$*, for all* $\mathcal{S}$*.*

2. *Given its institutional structure, the government selects a policy* $(\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S}))$ *subject to the government budget constraint* (19)*, for all* $\mathcal{S}$*.*

3. *All markets clear (goods, money, bonds), for all* $\mathcal{S}$*.*

The conduct of monetary policy determines what the treasury takes as given in choosing its policy, as detailed below.[25] Also, we characterize equilibria for given θ, share of government debt held by domestic agents, as its value is not pinned down in equilibrium.[26]

[24] Embedded in (18) is an interaction between inflation expectations, that determines real money holding $m(\mathcal{S}_{-1})$, and realized inflation. This element will give rise to strategic interactions between expected inflation and delivered inflation, an important ingredient to equilibria arising under discretion, see Section 4.

[25] Aguiar, Amador, Farhi, and Gopinath (2013) and Corsetti and Dedola (2016) study discretionary monetary authorities. Our analysis also highlights particular forms of commitment by the central bank as well as the role of reputation forces to implement policies without commitment.

[26] Again, this modeling feature guarantees that our results are not sensitive to the distribution of debt ownership.

Note that a market shutdown, where investors would charge a prohibitive interest rate to hold public debt, would not be consistent with the bond market clearing condition. Hence, throughout the upcoming discussions, we discard the possibility that debt does not have value.

3 Monetary Interventions under Commitment

This section studies the interaction of monetary interventions and debt fragility in a setting where the central bank is endowed with a commitment technology. Two cases are considered.

The first is a regime of strict inflation target, a common form of intervention. This discussion highlights the origins of debt fragility in our model and shows that inflation targeting alone does not eliminate this form of strategic uncertainty. Overcoming debt fragility requires more than a simple form of commitment by the monetary authority.

The second case enriches the policy to allow for state dependent interventions, while maintaining an inflation target on average. We argue that this second policy eliminates debt fragility. Further, it is consistent with the central bank's mandate to anchor price expectations.

As this policy requires partial commitment by the monetary authority, Section 4 provides conditions such that this outcome can be achieved without commitment as long as reputation effects are strong enough.

3.1 Strict Inflation Targeting

Under *"strict inflation targeting"*, or *"sit"*, the monetary authority is endowed with a commitment technology and is bound to deliver unconditionally an inflation target. We find that under this arrangement, debt valuations are sensitive to investors sentiment, as in the real economies of Calvo (1988) and Cooper (2012). The intuition behind this result is that a strict inflation target turns a nominal bond into a real debt contract.

Specifically, the central bank commits to an (inverse) inflation target $0 < \tilde{\pi}^* \leq 1$ and delivers it by printing money. By doing so, the central bank does not accommodate productivity shocks nor does it respond to sunspots. Revenue from seignorage is transferred to the treasury, which in turn decides to repay or default on its outstanding debt obligation. Though the central bank has some commitment power, the treasury continues to operate under discretion.

Formally, the policy of the central bank is:

$$\tilde{\pi}(\mathcal{S}) = \tilde{\pi}^* \qquad \forall \mathcal{S}. \tag{20}$$

As the central bank is bound to deliver its target $\tilde{\pi}^*$, agents' expectations are $\tilde{\pi}^e = \tilde{\pi}^*$.[27] In a stationary equilibrium, there is a stationary rate of money creation, σ^*, directly linked to the target inflation: $\frac{1}{1+\sigma^*} = \tilde{\pi}^*$. Revenue from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \nu^p m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})} \right) = \nu^p \tilde{\pi}^* (1 - \tilde{\pi}^*), \tag{21}$$

[27] In particular, if there is default, the monetary authority prints money and transfers it to old agents to meet this target.

as $m = m_{-1} = \tilde{\pi}^e = \tilde{\pi}^*$.[28] Within this monetary set-up, the government budget constraint under repayment becomes:

$$(1+i)\tilde{\pi}^* b = A^2(1-\tau)\tau + \nu^p \tilde{\pi}^*(1-\tilde{\pi}^*). \tag{22}$$

To derive that self-fulfilling debt crisis can arise under this monetary regime, we establish the existence of several interest rates that solve investors pricing equation (9). To do so, we first verify that the default decision has the following monotonicity property: if the treasury defaults for a given realization of technology $\bar{A}$, then it would default for any lower realization $A \leq \bar{A}$.

Lemma 1. *Given a level of real obligations $(1+i)\tilde{\pi}^* b$, there is a unique $\bar{A}(i) \in [A_l, A_h]$ such that if $A \leq \bar{A}(i)$, then the treasury defaults on its debt. Otherwise it repays its debt.*

Proof. See Appendix 6.2 ∎

From this result, the probability of default P^d becomes $F(\bar{A}(i))$. Altogether, an interest rate for the government debt solves:

$$(1+i)\tilde{\pi}^* \Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{23}$$

This equation may have several solutions, stemming from the interplay between beliefs of investors, probability of default and best-response of the government. Default arises both because of fundamental shocks (low A) and strategic uncertainty: the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability. It forms the basis for multiple valuations of government debt.

Lemma 2. *For any inflation target $0 < \tilde{\pi}^* \leq 1$, there are multiple interest rates that solve the no-arbitrage condition (23), including the risk-free rate and at least one equilibrium with a positive probability of default.*

Proof. See Appendix 6.3 ∎

This Lemma makes clear that the price of sovereign debt given by (23) is not uniquely determined. Under Assumption 3, there is always an equilibrium with certain repayment, where the nominal interest rate, noted $\underline{i}$, satisfies $(1+\underline{i})\tilde{\pi}^* = R$. Alternatively, investors expect default to occur with positive probability, and buy public debt only if it carries a higher interest rate $i > \underline{i}$. Given the increased burden of debt repayment, the treasury is induced to default for low realization of productivity A, which is consistent with the original beliefs of investors: there is $\bar{A} \in (A_l, A_h)$ and $i > \underline{i}$ that satisfy the no-arbitrage condition with state contingent default.

Lemma 2 provides the basis for the existence of a SREE in which sunspots matter under a regime of strict inflation target, i.e. the value of government debt is dependent upon the beliefs of investors. In equilibrium, there are sunspot dependent variations in employment, output and consumption.

[28] Revenue from seignorage is maximized at $\tilde{\pi}^L \equiv \frac{1}{2}$ which is the top of the seignorage "Laffer curve". At $\tilde{\pi}^* > \tilde{\pi}^L$, a reduction in $\tilde{\pi}^*$ (i.e. an increase in the rate of inflation) will increase revenue. The determination of the optimal inflation target $\tilde{\pi}^*$ is not part of the present analysis. The model could provide a positive theory of inflation, where the inflation target would be set to minimize distortions associated to tax revenue. Given the Laffer curve property of seignorage, any inflation target $0 < \tilde{\pi}^* < \tilde{\pi}^L$ is inefficient, but this does not affect the essential results regarding debt fragility.

Proposition 1. *Under "sit", for any $0 < \tilde{\pi}^* \leq 1$, there is a SREE with the following characteristics:*

1. *If $s_{-1} = s^o$, the government security is risk free and the treasury reimburses with probability 1.*

2. *If $s_{-1} = s^p$, the interest rate incorporates a risk-premium and the treasury defaults on its debt with positive probability.*

Proof. The characterization of the SREE directly comes from Lemma 2 and the existence of several interest rates compatible with the no-arbitrage condition in equilibrium. We describe the optimal behavior of agents consistent with the equilibrium definition.

As $\tilde{\pi}^e = \tilde{\pi}^* \in (0, 1]$, poor agents save only with money holding and wealthy young agents invest in intermediated claims. Indeed, consider a young household with productivity z. It can either save with money holding or via the financial sector, incurring the fixed cost Γ.

If it chooses to hold money, its labor supply when young is $n = z\tilde{\pi}^e$, its real demand for money holding is $zn = z^2\tilde{\pi}^e$ and the net expected contribution to consumption: $\big(z\tilde{\pi}^e\big)^2$. If it chooses the intermediated savings, its labor supply when young is $n = Rz$, its savings net of the intermediation cost is $Rz^2 - \Gamma$ and the net expected contribution to consumption: $R(Rz^2 - \Gamma)$. Hence, intermediated saving dominates money holding if and only if:

$$z^2 > \frac{R\Gamma}{R^2 - (\tilde{\pi}^e)^2}, \tag{24}$$

which is true for any $\tilde{\pi}^e \in (0, 1]$ as long as Assumption 1 holds. An aggregate fraction $\theta \in [0, 1]$ of the government security is held by domestic wealthy agents.

If $s_{-1} = s^o$, then young agents form expectations $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They supply labor accordingly. Consequently, the interest rate on debt satisfies the no-arbitrage condition (9) with $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i, seignorage revenue $\nu^p\tilde{\pi}^*(1 - \tilde{\pi}^*)$ and using Assumption 3, the optimal policy of the treasury is then to raise labor taxes τ for all A so as to satisfy its budget constraint and repay its debt.[29]

All markets clear. The money demand of the young poor agents is constant at $\tilde{\pi}^*$. The price level adjusts to ensure market clearing. From this, $\tilde{\pi}^* = \frac{1}{1+\sigma^*}$. In this equilibrium, inflation targeting and setting fixed money growth rate are equivalent. Given the no-arbitrage condition, the bond market clears assuming that foreign lenders have enough endowment to buy the government debt not purchased by domestic rich agents.

For the case $s_{-1} = s^p$, we outline only differences with the previous case. From Lemma 2, there is an interest rate i that carries a risk premium and satisfy the no-arbitrage condition, such that $(1 + i)\tilde{\pi}^* > R$. Young agents form expectations $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They price government debt according to $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i and seignorage revenue $\nu^p\tilde{\pi}^*(1-\tilde{\pi}^*)$, there is a unique threshold $\bar{A}(i)$ such that the optimal policy of the treasury is to raise labor taxes τ for all $A \geq \bar{A}(i)$ to satisfy the budget constraint and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F\big(\bar{A}(i)\big)$. ∎

Proposition 1 makes clear that debt fragility, identified in real economies extends to economies with nominal debt. Seignorage does reduce the real debt burden left to the fiscal authority, but without eliminating the underlying

[29] Relaxing Assumption 3 and allowing a fundamental equilibrium with positive probability of default does not change the central result that several interest rates are compatible with the no-arbitrage condition.

strategic uncertainty. The choice of the inflation target does not allow the monetary authority to peg the real interest rate, which instead, continues to reflect investors' sentiments.

From a life-time perspective, the welfare $V^{sit}(\tilde{\pi}^*, p_o)$ of a given generation under *"strict inflation targeting"* is negatively related to the probability of pessimism. Using (13) and (42):

$$V^{sit}(\tilde{\pi}^*, p_o) = p_o\Big[\int_{A_l}^{A_h} W^r(A, \underline{i})dF(A)\Big] \\ + (1-p_o)\Big[\int_{A_l}^{\bar{A}} W^d(A,i)dF(A) + \int_{\bar{A}}^{A_h} W^r(A,i)dF(A)\Big] - \sum_{j\in\{p,w\}} \nu^j \frac{(n_y^j)^2}{2}. \tag{25}$$

The first term corresponds to the expected welfare of old agents under optimism, when the nominal interest rate $\underline{i}$ induces repayment for any realization of technology A. The second term is the expected welfare under pessimism, where the risk premium included in the nominal interest rate $i > \underline{i}$ leads the treasury to default for low realizations of technology A. Finally, as inflation expectations are anchored, the third term that captures young agents' disutility of labor is independent of the realization of the sunspot shock.[30]

Importantly, the welfare of a generation under *"sit"* is increasing in the probability of optimism p_o and decreasing with the nominal interest rate i associated with pessimism. Indeed, the equilibrium under optimism Pareto dominates the pessimistic outcome, and the higher the risk premium under pessimism, the lower is welfare. By making the sunspot binary (optimism or pessimism), we restrict attention to equilibria with potentially at most two levels of the nominal interest rate, but there could be more self-fulfilling levels of interest rates associated with different default thresholds. Allowing the sunspot variable to have more than two realizations could capture these outcomes, without changing the essential nature of the analysis.

Finally, Proposition 1 is stated for any level of inflation target $\tilde{\pi}^*$. This does not imply though that the equilibrium is independent of the inflation target. The inflation target will influence seignorage revenue, the nominal interest rate and the fiscal burden.[31]

3.2 "*Whatever it takes*" - State Contingent Interventions

Instead of imposing an inflation target, suppose the central bank chooses a state contingent inflation policy that alters the real debt burden and distributes resources from seignorage across states. As in Chari, Christiano, and Eichenbaum (1998) this is a one period commitment, allowing the monetary authority to announce a policy and implement it next period, contingent on the current state. By carefully choosing the distribution of realized future inflation, the central bank can provide a shield against debt fragility.

Consider a rule given by $\tilde{\pi}(A, s_{-1})$: the current gross inverse inflation rate depends on current productivity as well as the sunspot realization from previous period.[32] In the equilibrium constructed below, optimism is equivalent to an interest rate satisfying $(1+\underline{i})\tilde{\pi}^* = R$. Pessimism is associated with the belief of investors that the treasury will default in some states. This leads to a risk-premium to government debt, a situation to which the monetary

[30] Formally, $\tilde{\pi}^e = \tilde{\pi}^*$, $n_y^p = \tilde{\pi}^*$ and $n_y^w = Rz$ from (4) and (10).

[31] This is explained in details in the working paper, Camous and Cooper (2014). Similarly, the existence of a sunspot equilibrium does not involve conditions on the level of debt b beyond Assumption 3, but the properties of the equilibrium are sensitive to b.

[32] As described below, an alternative expresses monetary interventions solely as a function of the interest rate, not the sunspots. This is made clear in the discussion of the policy implementation.

authority can respond.

This rule is devised with a couple of key properties. First, to induce agents to hold money, the rule will deliver a target rate of inflation.[33] Second, it will support the fundamental equilibrium by using monetary tools to counter pessimistic expectations so that equilibria with strategic uncertainty no longer exist. In this way, the monetary authority responds to variations in current beliefs, reflected in the sunspot and the interest rate, by appropriately setting policy for the future.

We first describe the desired properties of this policy and derive its existence in Lemma 3. Then, we characterize the stationary equilibrium of the economy under $\tilde{\pi}(A, s_{-1})$ and argue that such monetary policy rule stabilizes debt valuations.

Suppose the central bank commits to $\tilde{\pi}(A, s^o) = \tilde{\pi}^*$ for all A: under optimism, the central bank delivers an inflation target, as in Section 3.1, independently of the realization of productivity A. When $s_{-1} = s^p$, the central bank implements a state dependent monetary policy. This policy satisfies two key properties. First, $\tilde{\pi}(A, s^p)$ meets the inflation target $\tilde{\pi}^*$ *on average*:

$$\int_A \tilde{\pi}(A, s^p) dF(A) = \tilde{\pi}^*. \tag{26}$$

Combined with the policy under optimism, $\tilde{\pi}(A, s^o) = \tilde{\pi}^*$, unconditional inflation expectations are anchored at $\tilde{\pi}^*$. Thus, the real money tax base is invariant, and the government budget constraint under repayment becomes:

$$(1+i)\tilde{\pi}(A, s^p)b = A^2(1-\tau)\tau + \nu^p \tilde{\pi}^* \big(1 - \tilde{\pi}(A, s^p)\big). \tag{27}$$

Second, $\tilde{\pi}(A, s^p)$ is designed to deter state contingent default. That is, given A, inflation and a monetary transfer, the treasury chooses to reimburses its debt obligations with probability 1.

Lemma 3 establishes that there is a monetary policy rule $\tilde{\pi}(A, s^p)$ that satisfies these two properties.

Lemma 3. *Given an inflation target* $0 < \tilde{\pi}^* \leq 1$*, there is a monetary policy rule* $\tilde{\pi}(A, s^p)$*, that satisfies the inflation target and deters state contingent default. Moreover,* $\tilde{\pi}(A, s^p) > 0$ *for all* A*, and is increasing in* A*.*

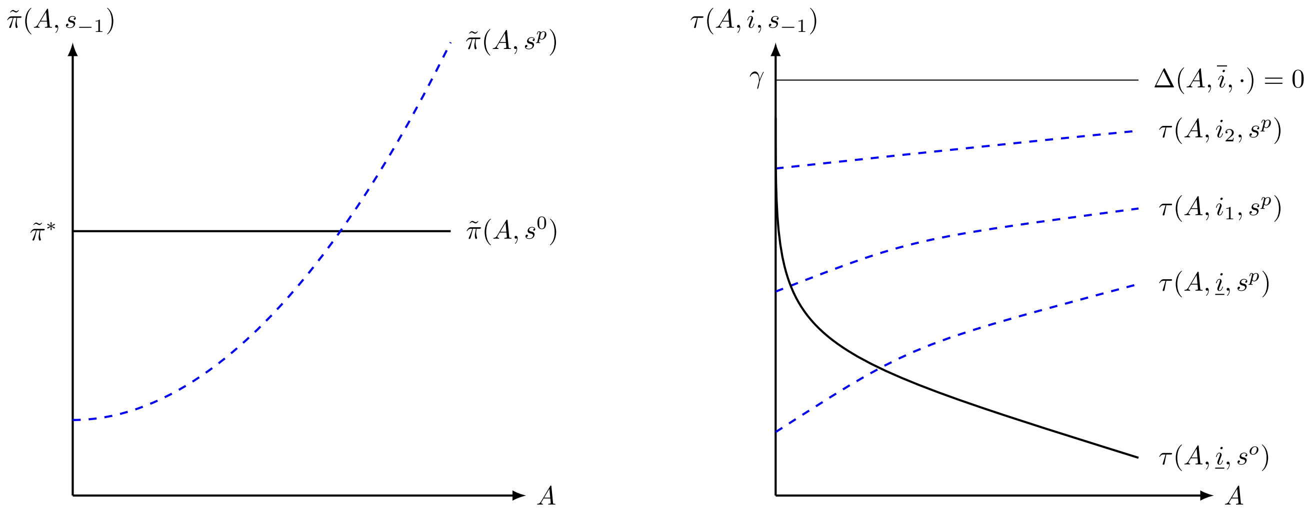
Proof. See Appendix 6.4 ∎

The lemma establishes two critical properties of the monetary intervention $\tilde{\pi}(A, s^p)$. First, for all A, $\tilde{\pi}(A, s^p) > 0$, which rules out demonetization of the economy and potential complete default via inflation. Second, if investors were pessimistic in the previous period, the monetary authority responds to variations in productivity: the rate of inflation is inversely related to current productivity. Specifically, when A is high, the rate of inflation is relatively low and fiscal policy, through the setting of tax rates, bears more of the burden of financing debt obligations. But during times of low productivity, when default is more likely, the monetary authority inflates the real value of debt, generates seignorage revenue and eases the incentives to default. The rule $\tilde{\pi}(A, s^p)$ is precisely devised to eliminate the incentives to default for all interest rate $i \geq \underline{i}$ such that debt has value.[34]

[33] The case where the central bank does not manage to anchor inflation expectations is covered in Section 4.2. It is shown then that self-fulfilling debt crisis are associated with self-fulfilling inflation.

[34] Formally, debt has value as long as $i \in [\underline{i}, \bar{i}]$. By construction, under $\tilde{\pi}(A, s^p)$, the treasury is indifferent between default and repayment at $i = \bar{i}$ and strictly prefers repayment for $\underline{i} \leq i < \bar{i}$ for all A. $i > \bar{i}$ is not compatible with bond market clearing.

Figure 1: State Dependent Monetary and Fiscal Policy



(a) Monetary Policy (b) Fiscal Policy

This graph represents policies on and off the equilibrium path. They depend on the sunspot, realized productivity and the nominal interest rate. The left panel draws the state dependent monetary policy to which the central bank commits. As stated in Lemma 3, it is independent of i. The right panel represents the induced fiscal policy, for various values of the interest rate: $\underline{i} < i_1 < i_2 < \bar{i}$, where $\bar{i}$ is an upper bound on the interest rate such that debt has value. The plain line $\Delta(\cdot) = 0$ represents the tax rate $\tau = \gamma$ such that the fiscal authority is indifferent between repayment and default. Under pessimism, the counter cyclical monetary intervention contains the incentives to default for low A as i increases, so that the fiscal authority repays its debt with probability 1.

Figure 1 displays the monetary policy rule and induced tax policy, as described in Lemma 3.[35] The left panel shows the monetary intervention, where, conditional on pessimism, the gross inverse inflation rate is increasing with A. The right panel illustrates the induced fiscal policy. Tax rates under repayment are set such that given the monetary intervention, the government budget constraint (27) is satisfied, hence the dependence of τ on i. An increase in productivity, coupled with a reduction in seignorage and inflation, leads to higher labor taxes, but the incentives to default are contained. This is true off the equilibrium path, where as $i \geq \underline{i}$ increases, tax rates increase for all A, but the treasury has no incentive to default. In this sense, the monetary authority *leans against the winds* of pessimism as well as those associated with low productivity.

Importantly, consider an economy where $\nu^p \approx 0$: seignorage revenue is negligible and still the intervention of the central bank deters debt fragility. Accordingly, the effectiveness of the monetary intervention relies on the unique capacity of the central bank to generate state contingent inflation and turn a non-state contingent nominal bond into a state contingent real asset.[36]

When the central bank commits to $\tilde{\pi}(A, s_{-1})$, there is a unique price for debt, namely the fundamental price under inflation targeting. That is, there is no sunspot equilibrium affecting the valuation of debt. Formally,

[35]The figure is obtained from a version of the model where $\theta = 0$, $\nu^p \approx 0$, so that $\tilde{\pi}^p_A = \frac{A^2 \tilde{\pi}^*}{\int_A A^2 dF(A)}$.

[36]Whenever all debt is held abroad, $\theta = 0$, the fiscal-monetary mix under $\tilde{\pi}(A, i, s^p)$ generates an allocation which coincides with the allocation under the optimal policy with commitment, as in Calvo and Guidotti (1993): the inflation tax should absorb all source of variation in technology A. Our analysis stresses that the benefits of countercyclical monetary policy extend to the prevention of self-fulfilling crises. Domestic holding of public debt modifies the repayment/default decision and hence the profile of $\tilde{\pi}(A, s^p)$. Finally, as the effectiveness of monetary interventions does not rely on the seignorage tax base, our results would not change if we were to consider a continuum of young agents, as in Freeman and Huffman (1991), rather than two types.

Proposition 2. *If the monetary authority commits to $\tilde{\pi}(A, s_{-1})$, with $\tilde{\pi}(A, s^o) = \tilde{\pi}^*$ and $\tilde{\pi}(A, s^p)$ given in Lemma 3, debt is uniquely valued and risk-free. Debt fragility is eliminated.*

Proof. Under Assumption 3, there is a risk-free outcome under strict inflation target $0 < \tilde{\pi}^* \leq 1$. Hence, there is an equilibrium nominal interest rate $\underline{i}$ under optimism that satisfies $(1+\underline{i})\tilde{\pi}^* = R$. Under pessimism, the monetary authority commits to $\tilde{\pi}(A, s^p)$ as defined in Lemma 3: this rule delivers inflation as a function of A, denoted $\tilde{\pi}^p_A$. We verify that under this rule, the best response of the treasury is to repay its debt for all A and that the only equilibrium interest rate is $\underline{i}$.

Suppose that investors believe the treasury will default with positive probability, and hence charge an interest rate $i > \underline{i}$ to buy government debt. Then the central bank will implement $\tilde{\pi}^p_A$. For some realization of A, the treasury will raise taxes and reimburse debt (otherwise debt would not have been issued in the first place, which is not consistent with the equilibrium definition). As established in Lemma 3, if the central bank implements $\tilde{\pi}^p_A$, then the treasury reimburses debt with probability 1, i.e. for all $A \in [A_l, A_h]$. This contradicts the initial hypothesis $i > \underline{i}$. Hence, under $\tilde{\pi}^p_A$, the nominal interest rate is $\underline{i}$:

$$(1+\underline{i}) \int_A \tilde{\pi}^p_A dF(A) = (1+\underline{i})\tilde{\pi}^* = R. \tag{28}$$

∎

This proposition makes clear that the commitment of the central bank rules out the effect of pessimism on the value of debt. The key to this result is the relaxation of the incentive to default by the fiscal authority through the erosion of the real return to debt in low productivity states.

The analysis has so far maintained Assumption 3, where the fundamental equilibrium is risk-free. It is straightforward to extend the analysis to a situation where the fundamental equilibrium is associated with a positive probability of default: self-fulfilling variations in the price of debt would reflect investors' sentiment under *"strict inflation targeting"*, as in Section 3.1; the monetary intervention characterized in Lemma 3 would only eliminate non-fundamental equilibria.[37]

Along the equilibrium path, from Proposition 2, only the fundamental price of debt will be observed. Though extraneous uncertainty may still exist, it will not be reflected in the equilibrium interest rate. With this in mind, it may be more natural to condition monetary interventions on interest rates. Along the equilibrium path, no actual intervention is needed, but the monetary authority stands ready to intervene in response to higher interest rates that reflect investors pessimism. This is, in effect, a threat of the monetary authority off the equilibrium path to intervene and support the fiscal authority to avoid default. In this case, the monetary authority commits to the following policy, labeled *"wit"*, for *"whatever it takes"*:

$$\begin{aligned} &\text{if } i = \underline{i}, &&\text{then} \quad \forall A \ \tilde{\pi}(A, i) = \tilde{\pi}^* \\ &\text{if } i \in (\underline{i}, \bar{i}), &&\text{then} \quad \forall A \ \tilde{\pi}(A, i) = \tilde{\pi}^p_A, \end{aligned} \tag{29}$$

where $\tilde{\pi}^p_A = \tilde{\pi}(A, s^p)$ is defined in Lemma 3 and $\bar{i}$ is the upper bound on the nominal interest rate such that debt

[37] Formally, if the fundamental equilibrium is associated with a default threshold $\bar{A}$, then $\tilde{\pi}(A, s^p) = \tilde{\pi}^*$ if $A < \bar{A}$ and, conditional on $A \geq \bar{A}$, $\tilde{\pi}(A, s^p)$ meets the inflation target and eliminates equilibria with default thresholds higher than $\bar{A}$.

has value.

With this implementation, the central bank commits to a strategy conditional on the nominal interest rate and ensures that private investors coordinate on the fundamental price of debt $\underline{i}$. In equilibrium, only the fundamental price of debt is observed and the central bank implements its unconditional inflation target.[38]

Under this rule, given an inflation target $\tilde{\pi}^*$, debt fragility is eliminated and the expected life-time welfare of private agents is given by:

$$V^{wit}(\tilde{\pi}^*) = \int_{A_l}^{A_h} W^r(A, \underline{i})dF(A) - \sum_{j\in\{p,w\}} \nu^j \frac{(n_y^j)^2}{2} \geq V^{sit}(\tilde{\pi}^*, p_o), \tag{30}$$

where $V^{sit}(\cdot)$ is the lifetime welfare under *"sit"*, defined in (25). The inequality is strict whenever the probability of optimism p_o is lower than 1.

4 Monetary Interventions under Discretion

The preceding analysis assumes the monetary authority is granted a commitment technology and argues that this power can eliminate debt fragility. This section relaxes the assumption of commitment, allowing the monetary authority to operate under discretion. In this case, inflationary expectations are determined through the equilibrium interaction of money demand and *ex post* optimal policy. Finally, additional costs of inflation are considered to study their impact on the credibility of monetary interventions. There are two main results.

First, there is a response to deviations from the *"wit"* policy that can support this intervention without commitment, as a sub-game perfect Nash equilibrium. The argument uses a version of the grim trigger strategy in repeated games, as in Rubinstein (1979).[39] There is a novel feature to this construction. The presence of strategic uncertainty through debt fragility makes deviations from policy *"wit"* costly and thus supports *"wit"* as an equilibrium outcome without commitment.

Second, absent reputation effects, debt fragility cannot be prevented by monetary interventions under discretion. Despite having the ability to stabilize fluctuations in debt prices, the monetary authority lacks the credibility to do so. Instead it relies as much as possible on the inflation tax. This leads to an adjustment of inflation expectations and generates both self-fulfilling debt crisis and inflationary policies under pessimism.

For this analysis, fiscal and monetary choices are both undertaken without commitment. As in the earlier analysis, monetary policy is implemented prior to fiscal policy. There is an asymmetry: reputation building is considered only for the monetary authority. This reflects our desire to study monetary interventions that stabilize debt markets otherwise subject to strategic uncertainty. If fiscal reputation effects were sufficiently strong, default would never occur, and the policy *"wit"* would not have been necessary in the first place.

[38] This approach is reminiscent of the analysis in Bassetto (2005). Indeed, committing to a specific strategy rather than to a policy rule allows the monetary authority to react to deviations from private agents and ensures a unique equilibrium outcome. In other words, committing to a strategy allows the monetary authority a second mover advantage in this dynamic game while anchoring expectations.

[39] Thus the analysis goes beyond standard Markov perfect equilibria to allow beliefs and actions to depend upon history.

4.1 Supporting *"wit"* through reputation

Can the central bank rely on its *reputation* to make *"wit"* credible? The monetary authority has an incentive to deviate from its stabilization policy to take advantage of the non-distortionary nature of the inflation tax and inflate beyond expectations. To counter this short-term gain, we construct an equilibrium in which any deviation from *"wit"* is met by a strict application of *"sit"*, described in Section 3.1. Hence, this analysis combines the two cases studied in Section 3 as the inflation target regime becomes a threat point to support the incentives to implement the state dependent policy that deters debt fragility. In particular, *"strict inflation targeting"* could be sustained with a similar reputational mechanism, where the threat point would be the demonetization of the economy.[40] In effect, our analysis investigates whether the central bank can credibly announce policy *"wit"*, i.e. anchor interest rates, by relying on its *institutional spine*, i.e. its long-acquired credibility to anchor inflation expectations.

The construction of the equilibrium goes as follows. The monetary authority seeks to implement *"wit"* described in (29). This generates lifetime welfare of $V^{wit}(\tilde{\pi}^*)$, given by (30). In a given state, the central bank could renege on its promise and consider any policy whatsoever. In the event of a deviation, *"wit"* is no longer credible. The monetary authority returns to its essential mandate of strict inflation targeting.

We consider two potential deviations from *"wit"*. The first occurs in normal times, i.e. for $i = \underline{i}$, when the central bank is supposed to follow its strict inflation target $\tilde{\pi}^*$. This is a check on incentives along the equilibrium path. The second occurs off the equilibrium path whenever the nominal interest rate is above $\underline{i}$ and the central bank implements the countercyclical policy rule $\tilde{\pi}(A, i) = \tilde{\pi}^p_A$, where $\tilde{\pi}^p_A$ is defined in Lemma 3 and illustrated in Figure 1. We do not consider deviations in inflation expectations alone, consistent with the idea that the central bank enforces *"wit"* by relying on its capacity to anchor expectations through an inflation target.[41]

For both of these deviations, we ask if the short term gains are offset by long term costs. Given monetary interventions, tax rates solve the government budget constraint (12).

The incentive for maintaining *"wit"* is characterized by the following difference in welfare:

$$\Delta(A, i, m_{-1}, p_o) = [W^{wit}(A, i, m_{-1}) - W^{dev}(A, i, m_{-1})] + \frac{\beta}{1-\beta}[V^{wit}(\tilde{\pi}^*) - V^{sit}(\tilde{\pi}^*, p_o)]. \quad (31)$$

Here $W^{dev}(A, i, m_{-1})$ is the current period payoff to old agents when a deviation occurs. The first term, $W^{wit}(A, i, m_{-1}) - W^{dev}(A, i, m_{-1})$ is, as we shall see, an immediate gain to old agents from deviating from *"wit"*.[42] The second term represents the long term losses from the deviation. If $\Delta(A, i, \tilde{\pi}^*, p_o) \geq 0$, then *"wit"* is incentive compatible in state (A, i). These terms are explained in turn.

Short Term Gains Any deviation from *"wit"* is through a policy that maximizes the static welfare of the current old. This policy minimizes distortionary taxation while relying more on seignorage and lower the real value of debt. Following Calvo (1978), we introduce an additional assumption to moderate the use of the inflation tax.

Assumption 4. *There is an arbitrary small lower bound on the inverse inflation rate:* $\tilde{\pi} \geq \underline{\tilde{\pi}}$.

[40] Indeed, the welfare in the real economy is lower than the welfare in the monetary economy under *"sit"*. We abstract from this reputational layer to preserve the clarity of the exposition.

[41] The case where the central bank looses its capacity to anchor inflation expectations is the purpose of the analysis next sub-section.

[42] As discussed below, this deviation has welfare consequences that are contained within a generation.

This is a constraint on the maximal rate of money creation and thus, in equilibrium, the minimal rate of inverse inflation, denoted $\underline{\tilde{\pi}}$. This assumption ensures that our results do not hinge upon the implausible capacity of the central bank to generate infinite inflation and eliminate the burden of debt.[43]

The short-term deviation from *"wit"* is characterized by the following state contingent static choices of monetary and fiscal policy.[44]

Lemma 4. *Given $\mathcal{S}$:*

1. *if debt is repaid, then*

   a. $\tilde{\pi}^r = \max\left\{\underline{\tilde{\pi}}, \Pi(\mathcal{S})\right\}$, *where* $\Pi(\mathcal{S}) = \frac{\nu^p m(\mathcal{S})}{\nu^p m_{-1}+(1+i)b}$,

   b. $\tau > 0$ *and solves the government budget constraint* (19) *if and only if* $\tilde{\pi}^r = \underline{\tilde{\pi}}$.

2. *if there is default, then* $\tau = 0$ *and* $\tilde{\pi}^d = \underline{\tilde{\pi}}$.

3. *there is default if and only if*

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} - (1+i)\underline{\tilde{\pi}}\theta b + T(\mathcal{S}, \underline{\tilde{\pi}}) > 0,$$

where τ solves the government budget constraint given $\tilde{\pi}^r = \underline{\tilde{\pi}}$ under repayment, and $T(\cdot)$ is the lump-sum transfer that implements $\underline{\tilde{\pi}}$ under default.

Proof. See Appendix 6.5 ∎

The payoff $W^{dev}(A, i, m_{-1})$ is thus given by (14), evaluated at the policy choice characterized in Lemma 4. In the event of default the inverse rate of inflation is set at $\underline{\tilde{\pi}}$, consistent with the rate that prevails when the government is indifferent between repayment and default.

Punishment The punishment to the deviation arises from the second term in (31), $\frac{\beta}{1-\beta}\left[V^{wit}(\tilde{\pi}^*) - V^{sit}(\tilde{\pi}^*, p_o)\right]$, where $\beta \in (0,1]$ is the rate at which the monetary authority discounts successive generations. Here the punishment for deviating from *"wit"* is the continuing operation of monetary policy under *"sit"*. In fact, resorting to the strict inflation target regime is a punishment precisely because it reopens the possibility of self-fulfilling debt crisis, as made explicit in (30): the lifetime welfare for each successive generation is $V^{sit}(\tilde{\pi}^*, p_o)$, given by (25), which depends positively on p_o, the probability of optimism. Hence, as $V^{wit}(\tilde{\pi}^*) = V^{sit}(\tilde{\pi}^*, 1)$, we have $V^{wit}(\tilde{\pi}^*) > V^{sit}(\tilde{\pi}^*, p_o)$ for $p_o < 1$.

Using this construction, the *"wit"* policy can be supported in an equilibrium without commitment if the costs of deviating from it are sufficiently high. Formally,

[43] Chari, Christiano, and Eichenbaum (1998) impose a similar restriction on the highest inflation regime that the central bank can implement. In the appendix of that paper, this restriction is rationalized by the presence of an alternative technology such that agents can bypass a cash-in-advance constraint. In effect, the return on this alternative technology pins down the worst sustainable equilibrium and thus a lower bound on the inverse rate of inflation. In our framework, the poor could store at a return of $r < 1$ instead of holding money and a parallel argument could be made.

[44] In this scenario, the central bank moves first and internalizes the reaction of the treasury, to either raise taxes and repay or default on debt.

Proposition 3. *If the probability of pessimism is sufficiently high and β close enough to unity, then the monetary authority will pursue the "wit" policy in all states, i.e.* $\Delta(A, i, \tilde{\pi}^*, p_o)$*, as defined in* (31) *with "sit" as a threat-point, is non-negative for all A and $i \in [\underline{i}, \bar{i}]$. Debt fragility is eliminated.*

Proof. Clearly $W^{wit}(A, i, \tilde{\pi}^*) - W^{dev}(A, i, \tilde{\pi}^*) \leq 0$ since the monetary authority deviating from its pre-announced policy could replicate *"wit"*. From Lemma 4, it will choose to generate some additional inflation to take advantage of the non-distortionary nature of this source of revenue. The inflation rate is higher than that under policy *"wit"* for any (A, i). Higher inflation relaxes the debt burden left to be serviced with distortionary taxation and unambiguously increases welfare. Also note that by construction *"wit"* deters state contingent default. A fortiori, no default is possible under the deviation.

As $V^{sit}(\tilde{\pi}^*, p_o)$ is increasing in p_o, $\Delta(A, i, \tilde{\pi}^*, p_o)$ is decreasing in p_o. So for low enough p_o, $V^{wit}(\tilde{\pi}^*) - V^{sit}(\tilde{\pi}^*, p_o)$ can be large. Further, for β close to unity, $\frac{\beta}{1-\beta}[V^{wit}(\tilde{\pi}^*) - V^{sit}(\tilde{\pi}^*, p_o)]$ can be arbitrarily large. Hence for p_o sufficiently small and β close enough to unity, $\Delta(A, i, \tilde{\pi}^*, p_o) > 0$ for all A and $i \in [\underline{i}, \bar{i}]$. ∎

The conditions for supporting *"wit"* have two components. The first is the usual condition that the monetary authority does not discount the future too heavily. The second is not standard and involves the strategic uncertainty of the model. A gain from *"wit"* is the elimination of debt crisis that do arise with probability $(1 - p_o)$ under the strict inflation target regime. As the probability of pessimism increases, the penalty associated with sticking to the inflation target regime is larger. Accordingly, the higher the risk of coordination failure under inflation targeting, the more credible it is for the central bank to promise to undertake *"wit"* to counter pessimistic beliefs.

This proposition nests two types of deviations from the equilibrium path. First, suppose investors in period $t-1$ believe in *"wit"*, charge the fundamental interest rate $\underline{i}$ and expect the unconditional inflation target $\tilde{\pi}^*$ in all states. Still in period t the monetary authority operating under *"wit"* can choose to deviate and implement a policy of the type characterized in Lemma 4. The gain from this is the use of the non-distortionary inflation tax, which is the highest for $A = A_l$. The cost is that *"wit"* is no longer credible. But the foundation of the monetary authority as following strict inflation targeting is not altered. If the conditions of Proposition 3 are satisfied, the monetary authority does not deviate along the equilibrium path.

Providing incentives for the monetary authority along the equilibrium path is necessary but not sufficient for *"wit"* to be incentive compatible. So, second, consider a deviation by investors in which they believe there is a positive probability of default implying $i > \underline{i}$. We maintain the integrity of the monetary authority and thus anchor inflationary expectations at $\tilde{\pi}^*$.[45] In this case, *"wit"* implements $\tilde{\pi}(A, i) = \tilde{\pi}^p_A$, where $\tilde{\pi}^p_A$ is characterized in Lemma 3. Here, the credibility of *"wit"* is not necessarily at stake for low values of A but precisely for high realization of the shock. Indeed, to deliver the inflation target *on average*, the central bank tightens monetary policy whenever the realization of technology is high. Still, if the conditions for Proposition 3 hold, the monetary authority will have an incentive to implement $\tilde{\pi}^p_A$ to preserve its reputation and continue with *"wit"*. In this case, the pessimism of investors is not warranted, whatever the realization of A.

Note that our construction considers the most profitable short-term deviation and a conservative long term punishment. Alternative specifications could make the possibility to sustain *"wit"* easier. For instance, the deviation

[45] As discussed earlier, policy *"wit"* applies only for interest rates below a level denoted $\bar{i}$. For pessimism sufficiently high so that $i > \bar{i}$, debt is not issued in the first place, so there is no credibility to evaluate.

from *"wit"* to *"dev"*, i.e. to full discretion, could be replaced by a deviation to *"sit"*, i.e. to the inflation target $\tilde{\pi}^*$. This would generate a smaller short term gain, making it easier to support *"wit"*.

Further, we are considering as a punishment a return to the *"strict inflation target"* regime. An alternative punishment arises if investors no longer even trust the monetary authority to meet the inflation target. In this case, the outcome reverts to full discretion, as studied in the next section. Once this outcome is characterized, we return to the theme of supporting *"wit"*.

***Ex post* Costs of Inflation** As a final point, it is of interest to understand the robustness of our results to the introduction of additional *ex post* costs of inflation.[46] Suppose the conditions for Proposition 3 hold. Along the equilibrium path, there is no inflation beyond the inflation target $\tilde{\pi}^*$. Off the equilibrium path, in the event of pessimism, the monetary authority is called upon to create state contingent inflation. Incentives to undertake this policy under *"wit"* may be undercut by additional costs of inflation. This is analyzed in detail in Appendix 6.8. With some modifications, a version of Proposition 3 is robust to the introduction of additional costs of *ex post* inflation.[47]

4.2 Full Discretion

If the conditions for Propositon 3 fail, then concerns about its reputation will not constrain the central bank. In this case, absent reputation effects, the monetary authority looses the incentives to implement *"wit"* and debt fragility remains.

An essential element of this environment is the interaction between expected and realized inflation and the associated effects on the demand for money and the nominal price of debt. The inflation target is no longer available to anchor inflation expectations. Specifically, if agents anticipate high inflation (low $\tilde{\pi}^e$), they would reduce labor supply in youth and their real money holdings m_{-1} accordingly. Similarly, investors in public debt require a higher nominal interest rate to be compensated for expected inflation.

To collect revenue from seignorage or inflate the real value of debt, the central bank then has to deliver a higher inflation rate (low $\tilde{\pi}$), consistent with the beliefs of agents. Hence, the capacity of the central bank to support a stressed fiscal authority is compromised by the interaction between expected and delivered inflation. This is where the endogeneity of money demand is particularly important in the analysis.

The formation of expectations by young agents reflects these *ex post* policy choices. Let $\tilde{\pi}^e(\mathcal{S})$ denote the expectation of future (inverse) inflation given the current state $\mathcal{S}$. Then the requirement of rational expectations is $\tilde{\pi}^e(\mathcal{S}) = E_{\mathcal{S}'|\mathcal{S}}\tilde{\pi}(\mathcal{S}')$ where the expectation is over the future state given $\mathcal{S}$. This condition will be used in the construction of equilibria under full discretion.

To characterize a SREE under this regime, we build upon the policy choices characterized in Lemma 4: we study the debt pricing dimension of the equilibrium and associated stationary inflation expectations. The equilibrium combines these essential elements.

[46] The baseline model already includes costs of *ex ante* inflation through the effects of the inflation tax on both the labor supply and money demand decisions of young poor workers. In addition, unanticipated inflation, such as that occurring through the deviation from *"wit"* to *"dev"*, reduces the distortionary labor tax and thus increases output and consumption. It also reduces the real value of debt.
[47] This analysis follows the suggestions of our referees.

First, we investigate whether debt fragility arises under this regime. We show that the multiplicity of interest rates consistent with the no-arbitrage condition (9) persists and interacts with inflation expectations.

Lemma 5. *Under full discretion, there are multiple interest rates, and associated inflation expectations, that solve the no-arbitrage condition (9).*

Proof. See Appendix 6.6 ∎

The key to that result is that inflation expectations and probability of default are jointly linked by the anticipation of the best response of the discretionary central bank. Whenever the equilibrium of the debt financing problem induces state contingent default, the inflation rate is maximal for all A. Hence, in the event of pessimism, young agents expect high inflation, i.e. $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$, which in turn prevents the central bank from inflating away the real value of debt.[48]

Under optimism, young agents anticipate the government will service its debt obligation for all $\mathcal{S}$. Given the bias toward inflationary financing of debt, what determines $\tilde{\pi}^e(s^o)$ is whether seignorage resource is enough to service principal and interest on debt for all (A, s). Formally, Lemma 4 established that with $s_{-1} = s^o$, given the real money tax base $\nu^p m_{-1} = \nu^p \tilde{\pi}^e(s^o)$, inflation delivered by the discretionary authority satisfies:

$$\tilde{\pi}^r(A, s, \cdot) = \max\left\{\frac{\nu^p \tilde{\pi}^e(s)}{\nu^p \tilde{\pi}^e(s^o) + (1+i)b}; \underline{\tilde{\pi}}\right\} \qquad \forall A \; \forall s \in \{s^o, s^p\}, \tag{32}$$

where the max operator captures whether seignorage resource is enough to service principal and interest on debt, and $\tilde{\pi}^e(s) = m(\mathcal{S})$ is the real money demand of current young agents, conditional on the realization of the current sunspot s.[49]

The following lemma establishes the existence of stationary inflation expectations under optimism that are consistent with the policy choice (32) of the monetary authority for all $b \in (0, \bar{b})$ and all $p_o \in [0, 1]$.

Lemma 6. *Under full discretion, there is a debt threshold $\hat{b} = \frac{\nu^p \underline{\tilde{\pi}}(1-\underline{\tilde{\pi}})}{R}$ such that:*

1. *If $0 < b < \hat{b}$, then $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ is consistent with the government choice $\tilde{\pi}^r(A, s, \cdot) > \underline{\tilde{\pi}}$, for all (A, s).*
2. *If $\hat{b} \leq b < \bar{b}$, then $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ is consistent with the government choice $\tilde{\pi}^r(A, s, \cdot) = \underline{\tilde{\pi}}$, for all (A, s).*

Proof. See Appendix 6.7. ∎

In the first case, the level of debt and inflation expectations are such that seignorage is sufficient to service debt for any realization of s. In the second case, the level of debt and inflation expectations are such that seignorage is not sufficient to service debt, and must be complemented with labor taxes for any realization of s.[50]

[48] This characterization does rely on the existence of the lower bound $\underline{\tilde{\pi}}$, but not its exact value. The assumption that the central bank cannot print an infinite amount of money and generate an unbounded level of inflation within period, is essential for debt to have value, especially under pessimism. Without this bound, debt would not be issued in the first place under pessimism, which again is not consistent with our equilibrium definition.

[49] If debt is not too large, then the inflation tax alone is sufficient to cover debt obligations: i.e. $\tilde{\pi}^r(A, s, \cdot) > \underline{\tilde{\pi}}$ for all (A, s) and $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$. In this case, $\tau(A, s) = 0$ for all (A, s). Else, the inflation tax will be maximal, $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, and supplemented by a labor tax. In both cases $D(A, s) = r$ for all (A, s).

[50] We do not impose further parametric restriction to ensure that $\hat{b} < \bar{b}$, where $\bar{b}$ is defined by Assumption 3. This requires the lower bound on productivity A_l or the cost of default γ to be high enough or the share of money holder ν^p to be low enough. If it were the case that $\hat{b} \geq \bar{b}$, then only case 1 of Lemma 6 would apply, our results would not be affected.

Lemma 6 is silent on the uniqueness of inflationary expectations under optimism. In fact, complementarities between expected and delivered inflation rates can give rise to multiple stationary levels of inflation expectations.[51]

The analysis has established the potential for multiple solutions to the debt valuation equation and the existence of inflation expectations consistent with monetary policy. Taken together, these elements create the basis for sunspot equilibria associated with the valuation of government debt under full discretion. Formally,

Proposition 4. *There is a SREE under full discretion with the following properties:*

1. *If $s_{-1} = s^o$, government debt is risk free as the treasury reimburses with probability 1, with either:*

   a. *if $0 < b < \hat{b}$, then $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ and for all (A, s) , $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$, $D(A, s, \cdot) = r$,*

   b. *if $\hat{b} \leq b < \bar{b}$, then $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ and for all (A, s), $\tilde{\pi}(A, s, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) > 0$, $D(A, s, \cdot) = r$.*

2. *If $s_{-1} = s^p$, the interest rate $i > \underline{i}$ incorporates a risk-premium. For all A, $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$. The treasury defaults on its debt for all $A < \bar{A}(i)$ where $\bar{A}(i) \in (A_l, A_h)$ and $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$.*

Proof. We describe the optimal behavior of agents consistent with the equilibrium definition. This proof builds on Lemmas 5 and 6, i.e. the existence of several interest rates and associated inflation expectations consistent with the equilibrium definition.

If $s_{-1} = s^o$, then by Assumption 3, debt is risk free. Two cases need to be distinguished, as established in Lemma 6. If $b < \hat{b}$, then inflation expectations under optimism $\tilde{\pi}^e(s^o)$ allow seignorage resource to be sufficient to service principal and interest on debt. Young agents form expectations of no default and $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$. They supply labor accordingly, young agents with low productivity save with money, young rich agents save via intermediated claims; the interest rate i on the government security satisfies the no-arbitrage condition (9) with a zero probability of default, i.e. $P^d = 0$, and $\tilde{\pi}^e(s^o)$. The optimal policy of the government is then to set for all A, all s, $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$ and repay the debt.

On the other hand, if $\hat{b} \leq b < \bar{b}$, then there is an equilibrium with $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, seignorage resource is not sufficient and taxes need be raised to service debt. Using Lemma 4 and Assumption 3, for all A, all s, $\tilde{\pi}(A, s, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, s, \cdot)$ solves the government budget constraint (19) and debt is repaid. Accordingly, young agents form expectations $P^d = 0$, $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, the government security is priced according to (9). In both cases, all markets clear.

For $s_{-1} = s^p$, we detail only the differences with the previous case. Independently of the level of b, young agents form expectations in which there is a positive probability of default, i.e. $P^d > 0$, and $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$. The government security is priced accordingly. Given i and seignorage revenue $\nu^m \underline{\tilde{\pi}}(1 - \underline{\tilde{\pi}})$, there is a unique threshold $\bar{A}(i) > A_l$ such that the optimal policy is to raise labor taxes τ for all $A \geq \bar{A}(i)$ so as to satisfy the budget constraint (19) and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F(\bar{A}(i))$. ∎

Does full discretion in monetary policy provide a shield against debt fragility? Can the government inflate the real value of debt and generate additional resources to service its debt? When the central bank looses the ability to anchor inflation expectations, the answer is negative. As the proposition makes clear, this result does not hinge upon a particular inflation ceiling $\underline{\tilde{\pi}}$.[52] Indeed, under pessimism, the interplay between inflation expectations and

[51]This possibility is not explored further as our results are independent of this form of multiplicity.

[52]Specifically, it holds for $\underline{\tilde{\pi}}$ arbitrarily close to 0, i.e. an inflation ceiling arbitrarily high.

real money tax base *corners* the central bank into a high inflation regime with no more capacity to inflate debt or provide additional resources to the treasury. Hence, under a regime of full discretion, the sunspot shock to investors confidence triggers a joint shift in inflation expectations and debt sustainability. This shift in inflation expectations is the driving force that neutralizes the discretionary strategy to print money and collect seignorage to service debt.

Finally, we return to the issue of supporting *"wit"*. Proposition 3 is stated for a punishment that entails the reversion to the inflation target regime. If instead, the punishment was to full discretion, then the continuation policy would be characterized by Proposition 4 and, for sufficiently low values of $\underline{\tilde{\pi}}$, the consequent welfare would be lower than that obtained under an inflation target.[53] Further, as debt fragility persists under full discretion, a high probability of pessimism reduces welfare. As noted earlier, this starker punishment makes it easier to support *"wit"*.

5 Conclusions

The goal of this paper is to determine whether monetary policy enhances or mitigates fiscal fragility. A committed central bank can deter debt fragility by designing a specific monetary policy rule. The policy requires the monetary authority to implement a countercyclical policy, that erodes the real value of debt and provides resources, through seignorage, in times of low productivity. By supporting the fiscal authority in these states, the incentive for default is eliminated. Sovereign debt is no longer subject to multiple valuations driven by investors' sentiments.

Absent commitment, if reputation effects are strong enough, this policy can be an equilibrium outcome even if the monetary authority acts solely under discretion. Interestingly, the credibility of this monetary strategy increases with the risk of self-fulfilling debt crisis.

Otherwise, debt fragility is not eliminated by monetary interventions. In particular, if the central bank is committed only to an inflation target, then debt fragility remains. At the other extreme, if the central bank is allowed complete discretion and discounts the future heavily, then inflating unconditionally the real value of debt is too tempting. High inflation is anticipated and priced by investors. Similarly, money demand adjusts so that seignorage does not provide substantial resources. Debt fragility remains.

One task ahead is the consideration of risk averse households. Doing so is of interest to study in more detail the distribution consequences of inflation as well as the effects of uncertainty on the monetary tax base.

[53] For sufficiently low values of $\underline{\tilde{\pi}}$, welfare is close to that of the non-monetary economy. In that case, welfare is unambiguously lower than under under *"sit"* for any inflation target $\tilde{\pi}^* \geq \underline{\tilde{\pi}}$.

6 Appendix

6.1 Welfare under Repayment and under Default

As explained in section (2.2), the repayment vs. default decision in this environment is a discrete choice that affects only the welfare of old agents. Hence, the welfare criteria of the treasury for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^p \Big[c_o^p(D) - \frac{n_o^p(D)^2}{2}\Big] + \nu^w \Big[c_o^w(D) - \frac{n_o^w(D)^2}{2}\Big]. \tag{33}$$

Using the labor supply policy functions from (4) and (10), we get the following consumption and labor supply vectors:

$$c_o^p(r) = An_o^p(r)(1-\tau) + m_{-1}\tilde{\pi}^r \qquad c_o^p(d) = An_o^p(d)(1-\gamma) + m_{-1}\tilde{\pi}^d + t$$

$$n_o^p(r) = A(1-\tau) \qquad n_o^p(d) = A(1-\gamma)$$

$$c_o^w(r) = An_o^w(r)(1-\tau) + (1+i)\tilde{\pi}^r b^w + Rk \qquad c_o^w(d) = An_o^w(d)(1-\gamma) + Rk + t$$

$$n_o^w(r) = A(1-\tau) \qquad n_o^w(d) = A(1-\gamma).$$

Using $\nu^w b^w = \theta b$, one can solve for k, the risk-free component of individual portfolio of wealthy agents from their budget constraint:

$$zn_y^w = Rz^2 = b^w + k + \Gamma \Rightarrow \ \nu^w Rk = \nu^w R\big(Rz^2 - \Gamma) - R\theta b. \tag{34}$$

We derive the expressions for $W^r(\cdot)$ and $W^d(\cdot)$:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^p m_{-1}\tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^w R(Rz^2 - \Gamma) \tag{35}$$

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^p m_{-1}\tilde{\pi}^d - R\theta b + \nu^w R(Rz^2 - \Gamma) + T(\cdot), \tag{36}$$

where τ solves the government budget constraint under repayment and $T(\cdot) = \nu^p m_{-1}\sigma\tilde{\pi}^d$ is a lump sum transfer that implements $\tilde{\pi}^d$ under default. Default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

6.2 Proof Lemma 1

Given a nominal interest rate i, the decision to repay or default on debt is given by $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$, where the relevant welfare criteria are given by (14) and (15) and the lump-sum transfer under default by $T(\tilde{\pi}^*) = \nu^p\tilde{\pi}^*(1-\tilde{\pi}^*)$. Hence, a point of indifference between default and repayment, $\bar{A}(i)$ solves:

$$\frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} = (1+i)\tilde{\pi}^*\theta b - \nu^p\tilde{\pi}^*(1-\tilde{\pi}^*), \tag{37}$$

where τ satisfies the government budget constraint (22). Denote by $G(A)$ the left side of (37). Clearly if $G(A)$ is monotonically decreasing in A, then the default decision satisfies the desired cut-off rule. Rewrite $G(A)$ as follow:

$$G(A) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \frac{A^2\tau(\tau-2)}{2}. \tag{38}$$

Using the government budget constraint, (38) rewrites:

$$G(A) = \frac{A^2\gamma(\gamma-2)}{2} - \left[(1+i)\tilde{\pi}^*b - \nu^p\tilde{\pi}^*(1-\tilde{\pi}^*)\right]\frac{(\tau-2)}{2(1-\tau)}. \tag{39}$$

The first term is negative since $\gamma < 1$. If seignorage revenue is enough to service debt, then no tax need be raised and $\bar{A}(i) = A_l$, by Assumption 2.[54] Otherwise, $(1+i)\tilde{\pi}^*b - \nu^p\tilde{\pi}^*(1-\tilde{\pi}^*) > 0$. Finally, we need to derive the monotonicity of $\frac{\tau-2}{1-\tau}$ with respect to A. Its derivative is:

$$\frac{-1}{(1-\tau)^2}\frac{d\tau}{dA} > 0, \tag{40}$$

which is positive since $\frac{d\tau}{dA} < 0$ for the lowest value of τ that solves the budget constraint. Overall, we have $G'(A) < 0$. Hence, the cut-off value $\bar{A}(i)$ is unique and default occurs if and only if $A \leq \bar{A}(i)$.

Note that if $\bar{A}(i) \leq A_l$, then debt is risk free. Finally, $\bar{A}(i) = A_h$ is inconsistent with bond market clearing.

6.3 Proof Lemma 2

In equilibrium, the debt financing problem is characterized by an interest rate i and a default threshold $\bar{A}$. Importantly, an equilibrium is such that *beliefs* of investors are consistent with the *best response* of the treasury.

Investors believe that the treasury defaults with probability $P^d = F(\bar{A})$. This belief induces $\bar{A}^b(i)$, the default threshold consistent with P^d:

$$(1+i)\tilde{\pi}^*\big(1 - F(\bar{A})\big) = R \Rightarrow \ \bar{A}^b(i). \tag{41}$$

Given i, the treasury decision to repay or default induces $\bar{A}^g(i)$, the realization of A for which the treasury is indifferent between default and repayment:[55]

$$\Delta(A,i) = W^d(A,i) - W^r(A,i) = 0 \Rightarrow \bar{A}^g(i). \tag{42}$$

An equilibrium requires $\bar{A}^b(i) = \bar{A}^g(i)$. The nominal interest rate i can takes value on $[\underline{i}, +\infty)$ where $\underline{i}$ is the nominal interest rate consistent with risk-free debt. Formally, it satisfies $(1+\underline{i})\tilde{\pi}^* = R$. We study the monotonicity properties of $\bar{A}^b(\cdot)$ and $\bar{A}^g(\cdot)$.

The default threshold $\bar{A}^b(i)$ induced by belief of investors has the following properties. First, $\bar{A}^b(\underline{i}) = A_l$: if investors charge $\underline{i}$, it means that they expect no default. Second, differentiating (41) with respect to $\bar{A}$ and i, one

[54] To see this, set $\theta = 0$, $m = 1$ and $\tilde{\pi}^r = 0$ in (14) and (15) and verify that $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) < 0$ under Assumption 2.

[55] Lemma 1 established that this threshold is unique. To determine $\Delta(A,i)$ from (14) and (15), set $\tilde{\pi} = m = \tilde{\pi}^*$ and set τ from (22) if the government decides to repay.

gets:

$$\frac{d\bar{A}^b(i)}{di} = \frac{\big(1 - F(\bar{A})\big)}{f(\bar{A})(1+i)} > 0, \tag{43}$$

since $f(\cdot) > 0$. Finally, $\lim_{i \to +\infty} \bar{A}^b(i) = A_h$.

The best response of the treasury to i is captured by $\bar{A}^g(i)$, the default threshold. Given Assumption 3, for low values of i, debt is risk free. Hence, there is $\epsilon > 0$ such that $\bar{A}^g(\underline{i} + \epsilon) = A_l$. Second, by differentiating (37) with respect to $\bar{A}$ and i, one gets:

$$\tilde{\pi}^* b\Big[\frac{1-\tau}{1-2\tau} - \theta\Big]di + \bar{A}\Big[(1-\gamma)^2 - \frac{(1-\tau)^2}{1-2\tau}\Big]d\bar{A} = 0. \tag{44}$$

The factor of di is positive since $\frac{1-\tau}{1-2\tau} > 1$ and the factor of $d\bar{A}$ is negative since $\frac{(1-\tau)^2}{1-2\tau} > 1$. Hence:

$$\text{if } \bar{A}^g(i) \in (A_l, A_h), \text{ then } \frac{d\bar{A}^g(i)}{di} > 0. \tag{45}$$

Finally, there is an upper bound $\bar{i}$ such that default occurs for all A if $i \geq \bar{i}$:

$$\forall i > \bar{i},\ \bar{A}^g(i) = A_h. \tag{46}$$

By continuity of the functions $\bar{A}^g(\cdot)$ and $\bar{A}^b(\cdot)$, there is a value $i > \underline{i}$ that satisfies $\bar{A}^g(i) = \bar{A}^b(i)$.

6.4 Proof Lemma 3

We adopt the following notations. Consider the central bank committing to a policy contingent on A, noted $\tilde{\pi}_A$, and such that $\int_A \tilde{\pi}_A dF(A) = \tilde{\pi}^*$. Given $m_{-1} = \tilde{\pi}^e(\cdot) = \tilde{\pi}^*$, where $\tilde{\pi}^*$ is the inflation target of the central bank, the discretionary default decision of the treasury is captured by:

$$\begin{aligned}\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A) &= W^d(\cdot) - W^r(\cdot) \\ &= \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} + \nu^p \tilde{\pi}^*(1 - \tilde{\pi}_A) - (1+i)\tilde{\pi}_A \theta b,\end{aligned} \tag{47}$$

where τ solves the government budget constraint given $\tilde{\pi}_A$:

$$G(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A) = A^2(1-\tau)\tau + \nu^p \tilde{\pi}^*(1 - \tilde{\pi}_A) - (1+i)\tilde{\pi}_A b = 0. \tag{48}$$

Moreover, in the economy with $\theta > 0$, default occurs for two reasons: either it is the best response of the treasury: $\Delta(\cdot) > 0$, or the fiscal capacity of the country cannot service debt, since $\tau \leq \frac{1}{2}$.

We show that there is a unique state dependent inflation policy $\tilde{\pi}(A, s^p)$, noted $\tilde{\pi}^p_A$ in the following developments, and an induced interest rate cut-off $\bar{i}$ such that the policy delivers the inflation target on average, and, if the central bank commits to $\tilde{\pi}^p_A$, then the fiscal authority services its obligation for all A for all $i < \bar{i}$.

We proceed in two steps: first we show that for any i^t, there is a unique policy $\tilde{\pi}_A(i^t)$ such that the treasury reimburses its debt if and only if $i < i^t$. Second, we show that there is a unique $\bar{i}$ such that $\tilde{\pi}_A(\bar{i})$ satisfies the inflation target. The desired policy is given by $\tilde{\pi}^p_A = \tilde{\pi}_A(\bar{i})$ for all A.

Part I. Consider a nominal interest rate i^t such that $1 + i^t > 0$ and a realization $A \in [A_l, A_h]$.

(i) The following elements establish that there is a unique inflation level $\tilde{\pi}_A(i^t)$ such that the fiscal authority is indifferent between repayment and default. First, there is an inverse inflation rate $\tilde{\pi}^1_A(i^t)$ such that debt is serviced with no taxes on labor income.

$$G(A, i^t, \tilde{\pi}^*, \tau, \tilde{\pi}^1_A(i^t)) = 0 \Rightarrow \tau = 0. \tag{49}$$

In this case, using Assumption 2, $\Delta(\cdot) < 0$. Using the government budget constraint with $\tau = 0$, one gets:

$$\tilde{\pi}^1_A(i^t) = \frac{\nu^p \tilde{\pi}^*}{\nu^p \tilde{\pi}^* + (1 + i^t)b} > 0. \tag{50}$$

Similarly, the central bank can set the inverse inflation rate to $\tilde{\pi}^2_A(i^t)$ so that if the treasury desires to service its debt, it has to set $\tau = \frac{1}{2}$. Formally:

$$\tilde{\pi}^2_A(i^t) = \frac{\frac{A^2}{4} + \nu^p \tilde{\pi}^*}{\nu^p \tilde{\pi}^* + (1 + i^t)b}. \tag{51}$$

Importantly, for any inflation rate between these two cases, the lower inflation, i.e. the higher $\tilde{\pi}_A$, the higher the tax rate to service debt. Formally, differentiating the government budget constraint w.r.t. τ and $\tilde{\pi}_A$:

$$\forall \tilde{\pi}_A \in [\tilde{\pi}^1_A(i^t), \tilde{\pi}^2_A(i^t)],\ \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^p \tilde{\pi}^* + (1 + i^t)b}{A^2(1 - 2\tau)} > 0. \tag{52}$$

Moreover, the lower inflation, i.e. the higher $\tilde{\pi}_A$, the higher the value of $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$:

$$\frac{d\Delta(\cdot)}{d\tilde{\pi}_A} = \frac{1 - \tau}{1 - 2\tau}(\nu^p \tilde{\pi}^* + (1 + i^t)b) - (\nu^p \tilde{\pi}^* + (1 + i^t)\theta b) > 0, \tag{53}$$

since $\frac{1-\tau}{1-2\tau} > 1$ for $\tau \in [0, \frac{1}{2})$.

Hence, there is a unique $\tilde{\pi}_A(i^t)$ that has the desired property to make the treasury indifferent between repayment and default. Especially,

- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}^2_A(i^t)\big) > 0$, then $\tilde{\pi}^1_A(i^t) < \tilde{\pi}_A(i^t) < \tilde{\pi}^2_A(i^t)$,
- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}^2_A(i^t)\big) \leq 0$, then $\tilde{\pi}_A(i^t) = \tilde{\pi}^2_A(i^t)$.

(ii) Next, we verify that for any $i < i^t$, the fiscal authority services its debt, otherwise for any $i > i^t$, it defaults. Given $\tilde{\pi}_A(i^t)$, we have:

$$\frac{d\Delta(\cdot)}{di} = A^2(1 - \tau)\frac{d\tau}{di} - \tilde{\pi}_A(i^t)\theta b = \frac{1 - \tau}{1 - 2\tau}\tilde{\pi}_A(i^t)b - \tilde{\pi}_A(i^t)\theta b > 0. \tag{54}$$

(iii) Also, we establish the following properties of $\tilde{\pi}_A(i^t)$:

$$\frac{d\tilde{\pi}_A(i^t)}{dA} > 0 \qquad\qquad \frac{d\tilde{\pi}^p_A(i^t)}{di^t} < 0. \tag{55}$$

If $\tilde{\pi}_A(i^t) = \tilde{\pi}^2_A(i^t)$, these properties are straightforward. In the case $\tilde{\pi}_A(i^t) < \tilde{\pi}^2_A(i^t)$, first differentiate the government budget constraint w.r.t. $(A, i, \tau, \tilde{\pi}_A)$ to get:

$$\frac{d\tau}{dA} = -\frac{2(1-\tau)\tau}{A(1-2\tau)} \qquad \frac{d\tau}{di} = \frac{\tilde{\pi}_A b}{A^2(1-2\tau)} \qquad \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^p\tilde{\pi}^* + (1+i)b}{A^2(1-2\tau)} \tag{56}$$

Then differentiate $\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A)$ w.r.t to its arguments and using the derivative of τ w.r.t $(A, i, \tilde{\pi}_A)$, one gets:

$$\Big[A(1-\gamma)^2 - A\frac{(1-\tau)^2}{1-2\tau}\Big]dA + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^p\tilde{\pi}^* + (1+i)b\big) - \big(\nu^p\tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0 \tag{57}$$

$$\Big[\frac{1-\tau}{1-2\tau}\tilde{\pi}_A b - \tilde{\pi}_A\theta b\Big]di + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^p\tilde{\pi}^* + (1+i)b\big) - \big(\nu^p\tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0. \tag{58}$$

Since $\frac{1-\tau}{1-2\tau} > \frac{(1-\tau)^2}{1-2\tau} > 1$ for all $0 \le \tau \le \frac{1}{2}$ and $0 \le \theta \le 1$, we get the desired results.

(iv) Finally, the limit behavior of $\tilde{\pi}_A(i^t)$ are derived from the inequality

$$\tilde{\pi}^1_A(i^t) < \tilde{\pi}_A(i^t) \le \tilde{\pi}^2_A(i^t), \tag{59}$$

which gives $\lim\limits_{i^t\to+\infty} \tilde{\pi}_A(i^t) = 0$ and $\lim\limits_{i^t\to-1} \tilde{\pi}_A(i^t) > 1$.

Part II. By applying the inflation target requirement (26), we show that there is a unique $\bar{i} > 0$ such that:

$$\int_A \tilde{\pi}_A(\bar{i})dF(A) = \tilde{\pi}^*. \tag{60}$$

Note $H(i) = \int_A \tilde{\pi}_A(i)dF(A)$, which is defined for all i such that $1 + i > 0$. The properties of $\tilde{\pi}_A(i)$ naturally convey to $H(i)$: $H(i)$ is strictly decreasing in i; $\lim\limits_{i\to+\infty} H(i) = 0$; $\lim\limits_{i\to-1} H(i) > 1$. Hence there is a unique $\bar{i}$ such that $H(\bar{i}) = \tilde{\pi}^*$.

Overall, the monetary policy rule $\tilde{\pi}(A, s^p)$ that meets the inflation target and deters state contingent default, exists, and satisfies:

$$\tilde{\pi}(A, s^p) = \tilde{\pi}_A(\bar{i}), \qquad \forall\ A. \tag{61}$$

6.5 Proof Lemma 4

If the treasury repays, it will first use the inflation tax to obtain revenue since this tax is not distortionary. It will use labor taxation only if needed to repay debt. Hence, if the real inflation tax base is large enough to service debt, then its labor tax policy is $\tau = 0$.

We derive first the condition under which seignorage alone is enough to service debt. Using (16), the government budget constraint under repayment is:

$$(1+i)\tilde{\pi}^r b = A^2(1-\tau)\tau + \nu^p m_{-1}\sigma^r\tilde{\pi}^r. \tag{62}$$

From this expression, if $\tau = 0$, then $(1+i)\tilde{\pi}^r b = \nu^p\tilde{\pi}^r m_{-1}\sigma$ implying $\sigma = \frac{(1+i)b}{\nu^p m_{-1}}$. Using (17), under repayment $\tilde{\pi}^r = \frac{m(\mathcal{S})}{m_{-1}}\frac{1}{1+\sigma}$, where m_{-1} is real money held by the old and $m(\mathcal{S})$ is the level of real money demand of the current young.[56] The resulting inverse rate of inflation is given by $\Pi(\mathcal{S}) = \frac{\nu^p m(\mathcal{S})}{(1+i)b+\nu^p m_{-1}}$. Hence, resource from seignorage is enough to service debt if $\Pi(\mathcal{S}) \geq \underline{\tilde{\pi}}$.

We next verify that $\Pi(\mathcal{S}) \geq \underline{\tilde{\pi}}$ implies the treasury chooses to service its debt rather than default, i.e. $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) < 0$. With $\tau = 0$, $\Delta(\cdot)$ is:

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} + \nu^p m_{-1}(\underline{\tilde{\pi}} - \tilde{\pi}^r) - (1+i)\tilde{\pi}^r\theta b + T(\mathcal{S}, \underline{\tilde{\pi}}). \tag{63}$$

Here $T(\cdot) = \nu^p m_{-1}\sigma^d\underline{\tilde{\pi}}$ is the lump-sum transfer that implements $\tilde{\pi}^d = \underline{\tilde{\pi}}$, with $\sigma^d = \frac{m(\mathcal{S})}{\underline{\tilde{\pi}} m_{-1}} - 1$. Also, as seignorage is sufficient to service principal and interest on debt, $(1+i)b = \sigma^r\nu^p m_{-1}$, with $\sigma^r = \frac{m(\mathcal{S})}{\tilde{\pi}^r m_{-1}} - 1$. Finally, by the definition of $\tilde{\pi}^D$, $\nu^p m_{-1}\tilde{\pi}^D(1+\sigma^D) = \nu^p m(\mathcal{S})$ for $D = r, d$. Rearranging (63), one gets:

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \nu^p m(\mathcal{S})\frac{\sigma^r}{1+\sigma^r}(\theta - 1). \tag{64}$$

This is negative by Assumption 2 as long as $\frac{m(\mathcal{S})\sigma^r}{1+\sigma^r}(1-\theta) < 1$. With $\theta \leq 1$ and $\sigma \geq 0$ under both optimism and pessimism, $\tilde{\pi} \leq 1$ so that $m(\mathcal{S})= \tilde{\pi}^e(\mathcal{S}) \leq 1$. Hence $\frac{m(\mathcal{S})\sigma}{1+\sigma}(1-\theta) < 1$. We get $\Delta(\cdot) < 0$, i.e. when seignorage is enough to service principal and interest, the treasury chooses not to default.

If resource from seignorage is not enough to service principal and interest on debt, then positive labor taxes are implemented: $\tau > 0$ if and only if $\underline{\tilde{\pi}} > \Pi(\mathcal{S})$. In this case, default is possible. Using these elements together with (14) and (15), one gets the expression for $\Delta(\cdot)$ stated in the Lemma.

6.6 Proof Lemma 5

Consider the debt pricing building block of the equilibrium. We show that there are several possible outcomes, and consistent with our equilibrium definition, these different outcomes are driven by the realization of the sunspot s_{-1}. Using Assumption 3 and Lemma 4, there is a risk-free equilibrium of the debt financing problem, with inflation expectations $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$.[57] This may arise with $\tau = 0$ and $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$ or, from Lemma 4, with $\tau > 0$ and $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$.

Suppose investors believe the treasury will default on its debt with positive probability. If the belief is self-fulfilling, then the optimal policy of the central bank is to set the inflation level to $\underline{\tilde{\pi}}$ for all A whether it reimburses its debt or defaults (see Lemma 4). Otherwise, resources from seignorage would be enough to cover principal and interest on debt, and default would be avoided for all realization of A. Hence, inflation expectations of agents are

[56] As seen in (4), the money demand of the young is driven by inflation expectations that are entirely independent of the current choices of the policy maker.

[57] In general $\tilde{\pi}^e(\mathcal{S})$ denotes expected (inverse) inflation. The notation $\tilde{\pi}^e(s)$ highlights the dependence of expectations on the sunspot, s. This is the expectation held by young agents regarding the future value of $\tilde{\pi}$. This value determines the labor supply and real money demand of young poor agents. It also influences the nominal interest rate, see (9).

consistent with the best response of the treasury at $\tilde{\pi}^e(s^p) = \underline{\tilde{\pi}}$. The no-arbitrage condition pricing public debt becomes:

$$(1+i)\underline{\tilde{\pi}}\Big(1 - F\big(\bar{A}(i)\big)\Big) = R, \tag{65}$$

where $\bar{A}(i)$, defined in Lemma 1, is the boundary of the default region given i.

From Lemma 2, we know that there are at least two interest rates i that are consistent with this equilibrium condition, one of which carries a risk-premium and induces the government to default for some realizations of A. Hence the initial pessimistic beliefs are self-fulfilling and support the existence of an interest rate that carries a positive probability of default.

6.7 Proof Lemma 6

The no-arbitrage condition gives: $(1+i)\tilde{\pi}^e(s^o) = R$. Accordingly, $\tilde{\pi}^e(s^o) \geq \underline{\tilde{\pi}}$ can be part of a stationary equilibrium under full discretion if and only if it satisfies:

$$\tilde{\pi}^e(s^o) = p_o \max\Big\{\frac{\nu^p\tilde{\pi}^e(s^o)}{\nu^p\tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)}b}; \underline{\tilde{\pi}}\Big\} + (1-p_o)\max\Big\{\frac{\nu^p\tilde{\pi}^e(s^p)}{\nu^p\tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)}b}; \underline{\tilde{\pi}}\Big\}, \tag{66}$$

where p_o is the probability of optimism.

First consider the situation in which seignorage alone is not sufficient to service principal and interest on debt. In this case, the government sets $\tilde{\pi}^r(s,\cdot) = \underline{\tilde{\pi}}$ for all s, and raises additional labor taxes. Agents form expectations accordingly and (66) writes:

$$\tilde{\pi}^e(s^o) = (1-p_o)\underline{\tilde{\pi}} + p_o\underline{\tilde{\pi}} = \underline{\tilde{\pi}}. \tag{67}$$

This case emerges whenever $\frac{\nu^p\underline{\tilde{\pi}}}{\nu^p\underline{\tilde{\pi}} + \frac{R}{\underline{\tilde{\pi}}}b} \leq \underline{\tilde{\pi}}$, which rewrites:

$$b \geq \hat{b} = \frac{\nu^p\underline{\tilde{\pi}}(1-\underline{\tilde{\pi}})}{R}. \tag{68}$$

Next, we show that whenever $0 < b < \hat{b}$, there is a level of inflation expectation under optimism, $\tilde{\pi}^e(s^o)$, such that seignorage alone is sufficient to service principal and interest on debt for all (A,s). (66) writes then:

$$\tilde{\pi}^e(s^o) = p_o\frac{\nu^p\tilde{\pi}^e(s^o)}{\nu^p\tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)}b} + (1-p_o)\frac{\nu^p\underline{\tilde{\pi}}}{\nu^p\tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)}b}. \tag{69}$$

Multiply both sides by $\nu^p\tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)}b$ and get:

$$\nu^p\tilde{\pi}^e(s^o)^2 - p_o\nu^p\tilde{\pi}^e(s^o) + Rb - (1-p_o)\nu^p\underline{\tilde{\pi}} = 0. \tag{70}$$

Hence, (69) has at least a positive solution if $b \leq b^{\alpha}$, where:

$$b^{\alpha} = \frac{p_o^2 \nu^p + 4(1-p_o)\nu^p \underline{\tilde{\pi}}}{4R}. \tag{71}$$

Under this condition, the solution to (70) that is necessarily positive[58] is given by:

$$\tilde{\pi}^e(s^o) = \frac{p_o + \sqrt{p_o^2 + 4(1-p_o)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^p}}}{2}. \tag{72}$$

This solution is compatible with (66) if it satisfies the following two conditions:

$$\frac{\nu^p \tilde{\pi}^e(s)}{\nu^p \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b} \geq \underline{\tilde{\pi}} \qquad \forall s \in \{s^o, s^p\}, \tag{73}$$

We verify that $b^{\alpha} \geq \hat{b}$ and that for all $b < \hat{b}$, when $\tilde{\pi}^e(s^o)$ is given by (72), then the conditions (73) are satisfied. Note $F(p_o) = 4R\big(b^{\alpha} - \hat{b}\big)$. Substituting and rearranging:

$$F(p_o) = p_o^2 \nu^p - p_o 4\nu^p \underline{\tilde{\pi}} + 4\nu^p \underline{\tilde{\pi}}^2 = \nu^p (p_o - 2\underline{\tilde{\pi}})^2 \geq 0, \tag{74}$$

which gives $b^{\alpha} \geq \hat{b}$.

Next, note $G(p_o, b) \equiv \tilde{\pi}^e(s^o)$, where $\tilde{\pi}^e(s^o)$ is given by (72). The feasibility condition (73) for $s = s^p$ then reads:

$$G(p_o, b) = \frac{p_o + \sqrt{p_o^2 + 4(1-p_o)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^p}}}{2} \geq \sqrt{\frac{Rb\underline{\tilde{\pi}}}{\nu^p(1-\underline{\tilde{\pi}})}}. \tag{75}$$

In this expression, the left side $G(p_o, b)$ is decreasing in b, whereas the right side is increasing in b; $G(p_o, 0) > 0$ and the right side is equal to 0 for $b = 0$; $G(p_o, \hat{b}) \geq \underline{\tilde{\pi}}$ and the right side is equal to $\underline{\tilde{\pi}}$, for $b = \hat{b}$. Hence for all $b < \hat{b}$, (75) is satisfied.

Finally, the feasibility condition (73) for $s = s^o$ requires $b \leq b^{\delta} = \frac{\nu^p}{4R}$ and:

$$\frac{1 - \sqrt{1 - 4\frac{Rb}{\nu^p}}}{2} \leq G(p_o, b) \leq \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^p}}}{2}. \tag{76}$$

Since $\underline{\tilde{\pi}}(1 - \underline{\tilde{\pi}}) \leq \frac{1}{4}$, we have $\hat{b} \leq b^{\delta}$. Note $\mathcal{B}_l(b)$ and $\mathcal{B}_u(b)$ the lower and upper bounds of this inequality.

$\mathcal{B}_l(b)$, is increasing in b, $\mathcal{B}_l(0) = 0$, $\mathcal{B}_l(\hat{b}) = \frac{1-\sqrt{1-4\underline{\tilde{\pi}}(1-\underline{\tilde{\pi}})}}{2} = \frac{1-\sqrt{(1-2\underline{\tilde{\pi}})^2}}{2} \leq \underline{\tilde{\pi}}$ for all $\underline{\tilde{\pi}} \in [0, 1]$. As $G(p_o, b)$ is decreasing in b and $G(p_o, \hat{b}) \geq \underline{\tilde{\pi}}$, we have that for all $b \in [0, \hat{b}]$, $G(p_o, b) \geq \mathcal{B}_l(b)$.

We finally verify that $G(p_o, b) \leq \mathcal{B}_u(b)$ for all $b < \hat{b}$. Taking the derivatives of $G(p_o, b)$ w.r.t. p_o:

$$\frac{dG(\cdot)}{dp_o} = \frac{1}{2}\Big(1 + \frac{p_o - 2\underline{\tilde{\pi}}}{\sqrt{p_o^2 + 4(1-p_o)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^p}}}\Big). \tag{77}$$

[58] The other solution to the polynomial can be both positive and feasible, hence there is possibly multiple stationary inflation regimes due to the Laffer curve property of seignorage.

If $p_o - 2\underline{\tilde{\pi}} > 0$, then $\frac{dG(\cdot)}{dp_o} > 0$. If $p_o - 2\underline{\tilde{\pi}} < 0$, then verify that $-1 \leq \frac{p_o - 2\underline{\tilde{\pi}}}{\sqrt{p_o^2 + 4(1-p_o)\underline{\tilde{\pi}} - 4\frac{Rb}{\nu^p}}} \leq 0$, so that again $\frac{dG(\cdot)}{dp_o} > 0$. Hence, for all $p_o \in [0,1]$, all $\underline{\tilde{\pi}} \in [0,1]$, all $b \in [0, \hat{b}]$:

$$G(p_o, b) \leq G(1, b) = \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^p}}}{2} = \mathcal{B}_u(b). \tag{78}$$

Overall, we have shown that for all $b \leq \hat{b}$, there is $\tilde{\pi}^e(s^o)$ that satisfies (72) and solves (66).

6.8 Credibility and Costs of Inflation

Following Barro and Gordon (1983) and Rogoff (1985), the benefits and costs of inflation are often modeled through a quadratic loss in the policymaker's objective function.[59] In these, the gains of (surprise) inflation arise from economic expansion above an inefficient level of output when realized inflation exceeds expectations. This gain is present in our model through an alternative mechanism: the reduction in distortionary taxes facilitated by unanticipated money creation.

Following this literature, additional costs of inflation to policymakers are modeled as:

$$\mathcal{C}(\tilde{\pi}) = \frac{\kappa}{2}(\tilde{\pi} - \tilde{\pi}^t)^2, \tag{79}$$

where $\kappa > 0$ parameterizes this loss. Here, $\tilde{\pi}^t$ is interpreted as a desirable inflation rate, whose specification is made clear below.[60]

This additional feature has no impact on the analysis under commitment. Especially, Lemma 3, i.e. the capacity of the central bank to eliminate debt fragility using *"wit"*, does not depend on the inclusion of additional costs $\mathcal{C}(\tilde{\pi})$. Incorporating costs along the lines of (79) could impact Proposition 3 and the credibility of *"wit"* off the equilibrium path, when investors hold pessimistic beliefs. The following discussion focuses on that situation.

Looking specifically at the incentives for *"wit"*, only the short term gain in (31) is impacted by this additional feature, as $V^{wit}(\cdot)$ and $V^{sit}(\cdot)$ are both associated with the implementation of $\tilde{\pi}^*$ on the equilibrium path: their difference is independent of $\mathcal{C}(\tilde{\pi})$. Thus, additional costs of inflation impact the one period gain to defection, revised as

$$[W^{wit}(A, i, m_{-1}) - \frac{\kappa}{2}\big(\tilde{\pi}^{wit}(A, i) - \tilde{\pi}^t\big)^2] - [W^{dev}(A, i, m_{-1}) - \frac{\kappa}{2}\big(\tilde{\pi}^{dev}(A, i) - \tilde{\pi}^t\big)^2]. \tag{80}$$

In (80), the first term is again the short-term return to following *"wit"* and the second term is the payoff to implementing *"dev"*. Here $\tilde{\pi}^{wit}(A, i)$ is the state contingent monetary policy prescribed by *"wit"*. Along these same lines, $\tilde{\pi}^{dev}(A, i)$ is the optimal monetary policy under the deviation incorporating the additional cost $\mathcal{C}(\tilde{\pi})$. Importantly, this is not the same as the policy characterized in Lemma 4 due to the inclusion of this additional *ex post* cost of inflation.

There are two interesting ways to model the target $\tilde{\pi}^t$ in (80). One, following Svensson (1997), is that it represents a state contingent inflation contract set with the monetary authority. A second is that this is simply the state independent inflation target, $\tilde{\pi}^*$. We discuss each in turn.

[59] Other related analysis, such as Calvo (1988) and Corsetti and Dedola (2016) adopt similar approaches.

[60] Some versions of this loss function set $\tilde{\pi}^t = 0$. Since our agents base decisions on $\tilde{\pi}$, the cost function is written in terms of the inverse inflation rate rather than the customary specification.

Following Svensson (1997), it is natural to set $\tilde{\pi}^t = \tilde{\pi}^{wit}(A, i)$. By this logic, there is no additional *ex post* cost to inflation if the monetary authority is simply following its pre-announced policy. Costs are only incurred in case of deviation from the contracted policy. From the envelope condition associated with $\tilde{\pi}^{dev}(\cdot)$, an increase in κ reduces the benefits of a deviation, making it easier to support *"wit"*. Consequently, the critical β to support *"wit"* from Proposition 3 is decreasing in κ.

If instead, $\tilde{\pi}^t$ is simply the fixed inflation target $\tilde{\pi}^*$, then the results depend on the magnitude of κ. For small values of κ, the deviation from *"wit"* is, as before, through high inflation to reduce the distortionary labor tax. Under *"wit"*, inflation varies but is centered around $\tilde{\pi}^*$. Overall, this additional cost of inflation makes it easier to support *"wit"*. For κ sufficiently large, the deviation from *"wit"* is valuable as it stabilizes inflation and thus avoids the large cost of deviating from the target that arises through the implementation of *"wit"*. Thus the introduction of this additional cost of inflation creates another benefit to the deviation, which makes it harder to support *"wit"*. As in the earlier discussion of Proposition 3, regardless of the source of the gain to deviation, it will be non-negative and finite. Hence it can always be dominated by the punishment for low p_o and β sufficiently large.

Redistribution There is another interpretation of (79) that is of particular relevance in our model: costly redistribution. Under the response to pessimism under *"wit"* and the deviation *"dev"*, *ex post* inflation redistributes between rich and poor. Of course, the direction of this redistribution depends on the rate of inflation. Specifically, replace (79) with:

$$\mathcal{C}(\tilde{\pi}) = \frac{\kappa}{2}(c^w - c^p)^2 = \frac{\kappa}{2}(m\tilde{\pi} - Rk)^2, \tag{81}$$

where κ is a loss to policymakers from inequality in consumption of wealthy c^w and poor c^p.[61] The second equality follows directly from the budget constraints of each type of agents. It highlights the dependence of the consumption of the poor on the (inverse) inflation rate and the independence of the consumption of the rich on inflation.[62]

The influence of redistributive concerns on the credibility of *"wit"* depends here again on the magnitude of κ. If κ is low, then inflation under *"dev"* is systematically very high, while it is centered around $\tilde{\pi}^*$ under *"wit"*, so that the latter policy is easier to support. But for κ sufficiently large, inflation under *"dev"* is contained, so that the additional costs are lower than under *"wit"*.

Uncertainty As noted by Barro and Gordon (1983), inflation costs could also derive from uncertainty.[63] In our model, if investors are pessimistic and the monetary authority follows a state contingent policy, then the real return to working of young poor agents is stochastic. With our current preference specification, this uncertainty has no effect. With risk aversion, the consequent output loss is an additional *ex ante* cost of following *"wit"*, associated to a reduction in the money tax base. This does not influence the capacity of the central bank to pursue *"wit"* and deter variations in debt valuations. Indeed, as noted in Section 3.2, the effectiveness of *"wit"* relies on its influence on the real return to debt, not the level of resources collected via seignorage.

[61] Here we retain the assumption of linear utility of private agents. Allowing for strict concavity would provide some foundations for κ and is on our research agenda. Here the objective is to understand how these concerns might impact the implementation of "*wit*" without commitment. This is aptly captured with $\kappa > 0$.

[62] Here we have set $\theta = 0$ to study the case where inflation has the largest impact on redistribution. Note that these redistributive concerns could provide a rationale for the inflation target $\tilde{\pi}^*$ in our earlier discussions.

[63] We appreciate the suggestion of a referee to note this effect in our model.

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