

Sovereign Debt Crisis

Module 3: Decomposing Sovereign Bond Yields

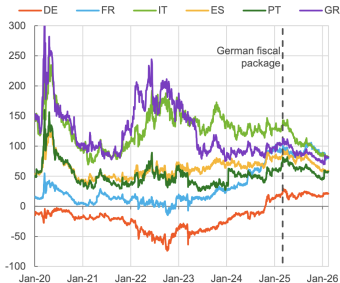
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Banque de France

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Motivation

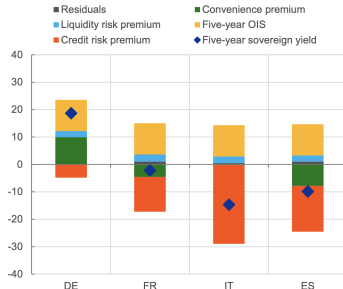
Spread of 10-year euro area government bond yields vs OIS rate
(basis points)



Sources: Bloomberg and ECB staff calculations.

Notes: The spread is the difference between individual countries' 10-year sovereign yields and the 10-year euro area OIS rate. German fiscal announcement refers to 5 March 2023. Latest observation: 16 February 2026.

Change in 5-year euro area sovereign yields since January 2025
(basis points)



Sources: Bloomberg, LSEG, CMA, Tradeweb and ECB calculations.

Notes: The decomposition of the DE, FR, IT, ES 5-year sovereign yields into its components: risk-free rate (5y OIS), default risk, redenomination risk, liquidity and convenience premia, is based on Corradin S. and Schwaab B. (2023). Credit risk premium includes default risk and redenomination risk premia. The change bars refer to the changes since 1 January 2025. Latest observation: 16 February 2026.

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Left: 10y sovereign spreads vs OIS. *Right:* Decomposition of yield changes since Jan 2025.

Source: ECB (2026), based on Corradin & Schwaab (2023) methodology.

A. Bond pricing basics & the yield decomposition problem

- Zero-coupon bonds, yields, the yield curve
- The Expectations Hypothesis
- Why it fails: the term premium
- For spreads: additional risk premia
- Corradin & Schwaab (2023): the frontier for the euro area

B. A teachable model: step-by-step derivation

- Working with spreads
- PCA on the spread curve
- Adding risk factors (credit risk, FX volatility)
- Affine no-arbitrage term structure model
- Decomposition into expectations & risk premium

C. Empirical results & policy implications

- EME application: Brazil, Colombia, Mexico
- EZ application: Corradin & Schwaab results

Part A

Bond Pricing Basics
& the Yield Decomposition Problem

Zero-coupon bonds and yields

A **zero-coupon bond** with maturity τ pays 1 at time $t + \tau$. Its price today:

$$P_t(\tau) = e^{-y_t(\tau) \cdot \tau}$$

The **continuously compounded yield**:

$$y_t(\tau) = -\frac{1}{\tau} \ln P_t(\tau)$$

- The **yield curve** is the function $\tau \mapsto y_t(\tau)$ at date t
- The **short rate** (policy rate): $r_t \equiv \lim_{\tau \rightarrow 0} y_t(\tau)$

In practice: overnight rate (Fed Funds, €STR, etc.)

Key intuition: A 10-year bond is a commitment to lend for 10 years. Its yield must reflect expectations about future short rates — plus possibly a premium for the risk of committing to that horizon.

The Expectations Hypothesis (EH)

Under EH, the long yield is the average of expected future short rates:

$$y_t(\tau) = \frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t[r_{t+j}]$$

No-arbitrage intuition: Two strategies for investing over τ periods:

- A.** Buy a τ -period zero-coupon bond \rightarrow locked-in return $y_t(\tau)$ per period
- B.** Roll over one-period bonds \rightarrow expected return $\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t[r_{t+j}]$

Risk-neutral investors equate the two \Rightarrow EH.

Numerical example: $r_t = 3\%$, $E_t[r_{t+1}] = 3.5\%$, $E_t[r_{t+2}] = 4\%$.

$$y_t(3) = \frac{3\% + 3.5\% + 4\%}{3} = 3.5\%$$

Upward-sloping curve \Rightarrow market expects rate hikes.

Why the Expectations Hypothesis fails

Empirical facts against EH:

- 1 Long bonds earn **excess returns** over short bills on average
- 2 The yield curve slope **predicts** excess bond returns — under EH it should not
- 3 The “term premium” is **time-varying**: it co-moves with the business cycle, inflation uncertainty, bond supply

The corrected decomposition:

$$y_t(\tau) = \underbrace{\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t[r_{t+j}]}_{\text{expectations component } EC_t(\tau)} + \underbrace{TP_t(\tau)}_{\text{term premium}}$$

The term premium compensates for:

- Interest rate (duration) risk
- Inflation risk
- Supply/demand effects (fiscal deficits, QE)

Key point: Neither $EC_t(\tau)$ nor $TP_t(\tau)$ is directly observed. The decomposition requires a **model**.

For sovereign spreads, the decomposition gets richer

The **spread** of country c over a benchmark (US Treasuries, German Bunds):

$$\text{spread}_t(\tau) = y_t^c(\tau) - y_t^{\text{benchmark}}(\tau) = \underbrace{EC_t(\tau)}_{\text{expected short-rate differential}} + \underbrace{RP_t(\tau)}_{\text{risk premium}}$$

The risk premium in the spread can reflect:

- **Credit/default risk**: probability of not being repaid in full
- **FX risk** (local-currency bonds): exchange rate uncertainty
- **Redenomination risk** (eurozone): possible euro exit
- **Liquidity risk**: compensation for illiquid markets
- **Segmentation/convenience**: regulatory frictions, home bias

⇒ Same observed spread can have very different interpretations depending on which component dominates.

Corradin & Schwaab (2023): the frontier for the euro area

CS achieve a granular decomposition for EZ sovereign yields:

$$y_t^c(\tau) = E_t[\text{rates}] + \text{Term P.} + \text{Default RP}^c + \text{Redenomination RP}^c \\ + \text{Liquidity RP}^c + \text{Segmentation}^c + u^c$$

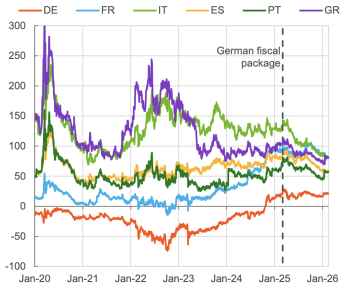
Identification of country-specific components:

- **Default RP**: CDS spreads under ISDA 2003 protocol
- **Redenomination RP**: ISDA basis (CT2014 – CT2003 CDS)
- **Liquidity RP**: KfW-Bund spread \times Tradeweb liquidity
- **Segmentation**: residual (convenience yield)

Application: DE, FR, IT, ES — 5-year bonds, daily, Jan 2015–Oct 2020.

CS decomposition in action (ECB, 2026)

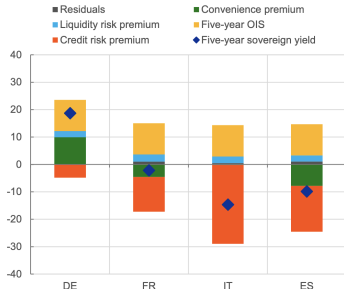
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Left: 10y sovereign spreads vs OIS. *Right:* Decomposition of yield changes since Jan 2025.

Source: ECB (2026), based on Corradin & Schwaab (2023) methodology.

CS headline result: PEPP announcement (March 18, 2020)

Italian 5y yields peaked at 196 bps, then **fell 78 bps** in 2 days:

Component	Δ (bps)
Default risk premium	-35
Segmentation premium	-16
Redenomination risk premium	-14
Expected rates & term premium	-8
Liquidity risk premium	-5
Total	-78

⇒ PEPP worked primarily through **credit and redenomination channels**, not through expected rates.

This is where the field is. Now let's build the tools to understand how.

Part B

Decomposing Sovereign Spreads: Step-by-Step Derivation

Based on Joslin, Priebsch & Singleton (*J. Finance*, 2014)
as applied in Burban and Golinski BdF (2025)

Step 0: Working with spreads

Object: For each date t and maturity τ , the zero-coupon **spread curve**:

$$s_t(\tau) \equiv y_t^{LC}(\tau) - y_t^{US}(\tau)$$

The spread **short rate**:

$$r_t^s \equiv r_t^{LC} - r_t^{US}$$

Why work with spreads rather than yields?

- Avoids joint estimation of two yield curves \Rightarrow fewer parameters
- Not subject to the zero lower bound (unlike US yields 2009–2015)
- Directly policy-relevant: the spread *is* the sovereign risk measure

Goal: Decompose $s_t(\tau) = EC_t(\tau) + RP_t(\tau)$

“How much of this spread is expected monetary policy divergence vs. risk compensation?”

The data



Figure 1. Sovereign spreads (%)

A. 3-month spreads

B. 10-year spreads



Source: Bloomberg, Gurkaynak, Sack and Wright (2006), computation from the authors. **Note:** Spreads are computed as the difference between the local currency bond yields and the U.S. Treasury yields. **Latest data point:** 30 July 2025.



Sovereign spreads vs US at 3-month and 10-year maturities. Brazil, Colombia, Mexico. Weekly, Jan 2009 – Jul 2025.

Source: Burban & Golinski, BdF (2025), Bloomberg, Gurkaynak et al. (2007).

Step 1: PCA on the spread curve

Problem: $N = 7$ maturities per date — high-dimensional. Need parsimony.

Solution: Collect spreads into a vector and run PCA:

$$S_t \equiv (s_t(\tau_1), \dots, s_t(\tau_N))' \in \mathbb{R}^N$$

Keep the first $K = 3$ principal components $X_t \in \mathbb{R}^3$:

$$S_t \approx a + WX_t$$

where $a \in \mathbb{R}^N$ (time mean), $W \in \mathbb{R}^{N \times 3}$ (PCA loadings).

Interpretation:

X_{1t} = **spread level** (parallel shift)

X_{2t} = **spread slope** (tilts short vs long end)

X_{3t} = **spread curvature** (bows the curve)

Empirical fact: First 3 PCs explain $> 99\%$ of spread variation (Burban & Golinski, Table 2B).

Step 2: Adding risk factors beyond the spread curve

The 3 PCs describe today's curve, but may not be enough to **forecast** future spreads.

If credit risk or FX vol help predict future spread movements *beyond what the current curve reveals*, they carry information that matters for the decomposition.

Constructing risk factors:

- 1 **Credit risk:** first PC of the CDS term structure

$$CR_t^{raw} = PC1(\text{CDS curve})$$

- 2 **FX risk:** 6-month implied FX volatility

$$FX_t = \text{FX implied vol}(6M)$$

- 3 **Orthogonalize** credit w.r.t. FX (avoid double-counting):

$$CR_t^{raw} = \alpha + \beta FX_t + CR_t, \quad E[CR_t \cdot FX_t] = 0$$

Define the additional factor vector: $U_t \equiv (CR_t, FX_t)' \in \mathbb{R}^2$

Step 2 (cont.): Do these factors actually help forecast?

Estimate a VAR(1) on the joint state (X_t, U_t) :

$$\begin{pmatrix} X_{t+1} \\ U_{t+1} \end{pmatrix} = c + \begin{pmatrix} A_{XX} & A_{XU} \\ A_{UX} & A_{UU} \end{pmatrix} \begin{pmatrix} X_t \\ U_t \end{pmatrix} + \eta_{t+1}$$

The key block: A_{XU}

- If $A_{XU} \neq 0$: credit risk and FX vol today help predict the **spread curve tomorrow**, beyond what today's curve already predicts
- These variables carry additional information — they belong in the model
- This is called **rejecting the spanning hypothesis**

Empirical result (Burban & Golinski, Table C1): For most countries, both credit risk and FX volatility coefficients are **statistically significant at 5%**.

Step 3: The affine no-arbitrage term structure model

Combine all factors into a single pricing state:

$$Z_t \equiv \begin{pmatrix} X_t \\ U_t \end{pmatrix} \in \mathbb{R}^5$$

Three key assumptions:

(i) **Affine short rate:**

$$r_t^s = \delta_0 + \delta_1' Z_t$$

(ii) **VAR(1) dynamics under the physical measure \mathbb{P} :**

$$Z_{t+1} = \mu + \Phi Z_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, I)$$

(iii) **Affine pricing** (from no-arbitrage):

$$s_t(\tau) = A(\tau) + B(\tau)' Z_t$$

where $A(\tau)$, $B(\tau)$ are maturity-dependent coefficients from no-arbitrage recursions.

Step 3 (cont.): Where do $A(\tau)$ and $B(\tau)$ come from?

Under the **risk-neutral measure** \mathbb{Q} , the state follows a different VAR:

$$Z_{t+1} = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} Z_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}$$

The difference (\mathbb{P} vs \mathbb{Q}) = the **market price of risk**.

No-arbitrage recursions (Riccati equations):

$$B(\tau) = (\Phi^{\mathbb{Q}})' B(\tau - 1) + \delta_1, \quad B(0) = 0$$

$$A(\tau) = A(\tau - 1) + B(\tau - 1)' \mu^{\mathbb{Q}} + \frac{1}{2} B(\tau - 1)' \Sigma \Sigma' B(\tau - 1) + \delta_0, \quad A(0) = 0$$

Key message: No-arbitrage + affine assumptions \Rightarrow spreads are **linear in the state**, with coefficients computable recursively.

Joslin-Singleton rotation: We can choose Z_t so that its first 3 elements = the observed PCs X_t . “Latent” factors become observable \Rightarrow simplifies estimation.

Step 4: The decomposition — the payoff

For each maturity τ :

$$s_t(\tau) = EC_t(\tau) + RP_t(\tau)$$

Expectations component: the spread under the EH (risk-neutral investors):

$$EC_t(\tau) \equiv \frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t^{\mathbb{P}}[r_{t+j}^s]$$

= average expected future short-rate differential (local vs US) over horizon τ .

Risk premium: the residual:

$$RP_t(\tau) \equiv \hat{s}_t(\tau) - EC_t(\tau)$$

= compensation for bearing risks (credit, FX, liquidity, etc.) beyond expected policy divergence.

Computing $EC_t(\tau)$ in closed form

We need $E_t^{\mathbb{P}}[r_{t+j}^s]$ for each j . Since $r_t^s = \delta_0 + \delta_1' Z_t$ and $Z_t \sim \text{VAR}(1)$:

VAR iteration (by induction): the j -step-ahead conditional mean is

$$E_t^{\mathbb{P}}[Z_{t+j}] = \Phi^j Z_t + \sum_{k=0}^{j-1} \Phi^k \mu$$

Proof. Base: $E_t[Z_{t+1}] = \Phi Z_t + \mu \checkmark$. Inductive step:

$$E_t[Z_{t+j+1}] = \mu + \Phi \cdot E_t[Z_{t+j}] = \Phi^{j+1} Z_t + \sum_{k=0}^j \Phi^k \mu \checkmark$$

Substituting into the short rate:

$$E_t^{\mathbb{P}}[r_{t+j}^s] = \delta_0 + \delta_1' \left(\Phi^j Z_t + \sum_{k=0}^{j-1} \Phi^k \mu \right)$$

Computing $EC_t(\tau)$: closed-form expression

Averaging over $j = 0, \dots, \tau - 1$:

$$EC_t(\tau) = \delta_0 + \delta_1' \left[\frac{1}{\tau} \sum_{j=0}^{\tau-1} \Phi^j Z_t + \frac{1}{\tau} \sum_{j=0}^{\tau-1} \sum_{k=0}^{j-1} \Phi^k \mu \right]$$

And the **risk premium**:

$$RP_t(\tau) = \hat{s}_t(\tau) - EC_t(\tau) = [A(\tau) + B(\tau)' Z_t] - EC_t(\tau)$$

Since both $\hat{s}_t(\tau)$ and $EC_t(\tau)$ are affine in Z_t , the risk premium is also **affine in the state** — it inherits a clean, interpretable structure.

Interpretation

Component	Formula	Captures
$EC_t(\tau)$	$\frac{1}{\tau} \sum E_t^{\mathbb{P}}[r_{t+j}^s]$	Expected policy-rate differential
$RP_t(\tau)$	$\hat{s}_t(\tau) - EC_t(\tau)$	Risk compensation
$s_t(\tau)$	$EC_t(\tau) + RP_t(\tau)$	Observed spread

The same 200 bps spread can mean very different things:

- A.** $EC = 180$ bps, $RP = 20$ bps \Rightarrow expected monetary policy divergence. Spread reflects **fundamentals**.
- B.** $EC = 40$ bps, $RP = 160$ bps \Rightarrow markets expect policy convergence, but demand heavy **risk compensation**. **Warning signal**.

Step 5: Estimation (overview)

Factor extraction method (Goliński & Spencer, 2024):

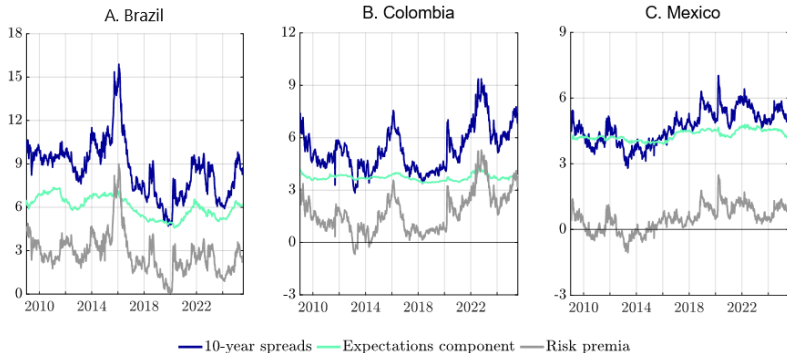
- ① Extract X_t via PCA (Step 1)
- ② Construct U_t from CDS and FX data (Step 2)
- ③ Estimate VAR(1) on $Z_t = (X_t', U_t')'$ by OLS $\Rightarrow \hat{\mu}, \hat{\Phi}, \hat{\Sigma}$
- ④ Estimate short-rate equation $r_t^s = \delta_0 + \delta_1' Z_t$ by OLS
- ⑤ Estimate risk-neutral parameters $(\mu^{\mathbb{Q}}, \Phi^{\mathbb{Q}})$ via cross-sectional fit of the affine pricing equation
- ⑥ Compute $EC_t(\tau)$ and $RP_t(\tau)$ from the closed-form expressions

Part C

Empirical Results
& Policy Implications

EME application: the decomposition in action

Figure 2. Decompositions of 10-year term spreads into expectations and risk premia



Note: Sample from January 2009 to July 2025.

Decomposition of 10-year spreads into expectations component and risk premia. Brazil, Colombia, Mexico. Jan 2009 – Jul 2025.

Source: Burban & Golinski, BdF (2025).

What drives spread variance? Credit risk vs. FX volatility

	Brazil	Colombia	Mexico
<i>Variance share of 10y spread:</i>			
Risk premium share	~ 50%	~ 81%	~ 52%
<i>Correlation of 10y RP_t with:</i>			
Credit risk	43%	37%	≈ 0
FX volatility	52%	71%	high

Key findings:

- Risk premium dominates long-term spread variance everywhere
- Brazil: credit risk and FX vol contribute roughly equally
- Colombia & Mexico: FX volatility is the dominant driver
- The expectations component is persistent and slow-moving (monetary policy cycle alignment with the Fed)

Back to the euro area: CS (2023) results

Italy & Spain: default and redenomination risk premia dominate

- Italian redenomination RP spiked to ~ 90 bps in mid-2018 (Lega/M5S coalition)
- Default RP = 45% of Italian “yield” during COVID (Jan–Jul 2020)

France & Germany: expected rates + segmentation dominate

- German segmentation premium ≈ -36 bps (most sought-after bond)
- French redenomination RP spiked ~ 30 bps before May 2017 election

Policy implications:

- PEPP (2020): reduced IT yields 78 bps, mostly via credit & redenomination channels
- EU fiscal announcements (Apr 2020): uniform yield reduction across countries
- The decomposition = the analytical backbone behind ECB yield monitoring (TPI, PEPP, “whatever it takes”)

Summary

- ① Sovereign yields/spreads embed **multiple unobservable components**: expected rates, term premium, credit risk, FX risk, liquidity, segmentation
- ② The **affine DTSM with unspanned risks** (Joslin et al., 2014) provides a tractable, closed-form decomposition:

$$s_t(\tau) = EC_t(\tau) + RP_t(\tau)$$

- EC_t : average expected future short-rate differential (from the VAR)
 - RP_t : residual risk compensation (driven by credit, FX, etc.)
- ③ **Empirically**: risk premia dominate long-term spread variance, but their composition varies (credit-driven in Brazil, FX-driven in Mexico, default+redenomination in Italy)
 - ④ This framework is the **analytical tool** behind central bank yield monitoring and policy response

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